XXII Meeting of Physics 2022

Black hole stability under odd-parity perturbations in Horndeski gravity

Jeferson Arroyo Cóndor*
*jeferson.arroyo.c@uni.pe, Universidad Nacional de Ingeniería

INTRODUCTION

The action of the most general scalar-tensor theory in four dimensions, having second-order field equations, is

$$\begin{aligned} S = \int \mathrm{d}^4x \Big[& K(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R \\ & + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\Big] \end{aligned} \tag{1}$$

The background space-time under consideration will be

$$\mathrm{d}\overset{0}{s}{}^2 = -A(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + C(r)\left(\mathrm{d}\theta^2 + \mathrm{sen}^2\,\theta\mathrm{d}\varphi^2\right) \tag{2}$$

PERTURBATION FORMALISM

We consider a background metric $\overset{0}{g}_{\mu\nu}$ and a perturbed metric $g_{\mu\nu} = \overset{0}{g}_{\mu\nu} + h_{\mu\nu}$. The 10 components of the perturbed metric $g_{\mu\nu}$ transform as scalar, vector or tensor. However, it can be shown that there are actually 7 scalar (even) and 3 vector (odd) perturbations. The 3 vector perturbations take the form

$$h_{0I} = \sum_{\ell m} h_0^{\ell m}(t,r)E_{IJ}\partial^J Y_{\ell m} \tag{3}$$

$$h_{1I} = \sum_{\ell m} h_1^{\ell m}(t,r)E_{IJ}\partial^J Y_{\ell m} \tag{4}$$

$$h_{IJ} = \frac{1}{2}\sum_{\ell m} h_2^{\ell m}(t,r)\left[E_I{}^K\nabla_{KJ}Y_{\ell m} + E_J{}^K\nabla_{KI}Y_{\ell m}\right] \tag{5}$$

For $\ell \geq 2$, we can to use Regge-Wheeler gauge, i.e., $h_2^{\ell m} = 0$.

SECOND ORDER ACTION $\ell \geq 2$

Expanding the action (1) to second order in perturbations and performing integration over the sphere (θ,φ), we find

$$S_{\text{odd}}^{(2)} = \int \mathrm{d}t\,\mathrm{d}r\,\mathcal{L}_{\text{odd}}^{(2)} \tag{6}$$

$$\mathcal{L}_{\text{odd}}^{(2)} = a_1 h_0^2 + a_2 h_1^2 + a_3\left[\dot{h}_1^2 + h_0'^2 - 2\dot{h}_1 h_0' + 2\frac{C'}{C}\dot{h}_1 h_0\right]$$

$$a_1 = \frac{\ell(\ell+1)}{4C}\left[\frac{\mathrm{d}}{\mathrm{d}r}\left(C'\sqrt{\frac{B}{A}}\mathcal{H}\right) + \frac{(\ell-1)(\ell+2)}{\sqrt{AB}}\right] \tag{7}$$

$$a_2 = -\frac{\ell(\ell+1)}{2}\sqrt{AB}\left[\frac{(\ell-1)(\ell+2)}{2C}\mathcal{G}\right] \tag{8}$$

$$a_3 = \frac{\ell(\ell+1)}{4}\sqrt{\frac{B}{A}}\mathcal{H} \tag{9}$$

$$\mathcal{F} = 2\left[G_4 + \frac{1}{2}B\phi'X'G_{5X} - XG_{5\phi}\right] \tag{10}$$

$$\mathcal{G} = 2\left[G_4 - 2XG_{4X} + X\left(\frac{A'}{2A}B\phi'G_{5X} + G_{5\phi}\right)\right] \tag{11}$$

$$\mathcal{H} = 2\left[G_4 - 2XG_{4X} + X\left(\frac{C'}{2C}B\phi'G_{5X} + G_{5\phi}\right)\right] \tag{12}$$

STABILITY ANALYSIS $\ell \geq 2$

The second order action can be reduced to the form

$$S_{\text{odd}}^{(2)} = \int \mathrm{d}t\,\mathrm{d}r\left[\alpha\dot{q}^2 + \beta q'^2 + \gamma q^2\right] \tag{13}$$

In order to avoid a ghost, we need to impose $\alpha > 0$, which means $\mathcal{G} > 0$ and the condition to avoid Laplacian instabilities is $\beta < 0$ or $\mathcal{F} > 0$. Carrying out certain operations, it is possible to obtain $\mathcal{S}_{\text{odd}}^{(2)}$ as

$$\frac{\ell(\ell+1)}{4(\ell+2)(\ell-1)}\int \mathrm{d}t\,\mathrm{d}r^*\left[\frac{\mathcal{F}}{\mathcal{G}}\dot{Q}^2 - \left(\frac{\mathrm{d}Q}{\mathrm{d}r^*}\right)^2 - VQ^2\right] \tag{14}$$

$$\begin{aligned} V = {} & l(l+1)\frac{A}{C}\frac{\mathcal{F}}{\mathcal{H}} - \frac{C^2}{4C'}\left(\frac{ABC'^2}{C^3}\right)' \\ & - \frac{C^2\mathcal{F}^2}{4\mathcal{F}'}\left(\frac{AB\mathcal{F}'^2}{C^2\mathcal{F}^3}\right)' - \frac{2A\mathcal{F}}{C\mathcal{H}} \end{aligned} \tag{15}$$

From the variation of the action (14), we find

$$-\frac{\partial^2 Q}{\partial t^2} + \frac{\mathcal{G}}{\mathcal{F}}\frac{\partial^2 Q}{\partial r^{*2}} - \frac{\mathcal{G}}{\mathcal{F}}VQ = 0 \tag{16}$$

Here we impose $\mathcal{H} > 0$, which gives positive squared propagation speeds along the angular direction. Asumming that

$$Q(t,r) = \sum_n e^{-\imath\omega_n t}\psi_n(r), \quad \mathrm{Im}(\omega_n) < 0 \tag{17}$$

$$\Rightarrow \left(-\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} + V\right)\psi_n = \frac{\omega_n^2}{c_r^2}\psi_n \tag{18}$$

Conclusion: It is found that Horndeski black holes are linearly stable under odd perturbations [1] if and only if: $\mathcal{F} > 0$, $\mathcal{G} > 0$, $\mathcal{H} > 0$ and $S(r_h) \geq 0$, $S(r_\infty) \leq 0$, where

$$S = \frac{\sqrt{AB}}{2}\left(\frac{C'}{C} + \frac{\mathcal{F}'}{\mathcal{F}}\right) \tag{19}$$

APLICATION TO GR

In this theory we have $K = 0$, $G_3 = 0$, $G_4 = 1/2$, $G_5 = 0$, then, $\mathcal{F} = \mathcal{G} = \mathcal{H} = 1 > 0$. Is easy to calculate that

$$S = \frac{1 - 2M/r}{r} \rightarrow S(r_h) = S(r_\infty) = 0 \tag{20}$$

Therefore the Schwarzschild black hole is stable, in addition,

$$V(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right] \tag{21}$$

is the Regge-Wheeler potential.

REFERENCES

[1] Apratim Ganguly, Radouane Gannouji, Manuel Gonzalez-Espinoza y Carlos Pizarro-Moya. «Black hole stability under odd-parity perturbations in Horndeski gravity». En: *Class. Quant. Grav.* 35.14 (2018), pág. 145008. DOI: 10.1088/1361-6382/aac8a0.