QCD Trace Anomaly at the Interior of Twin Neutron Stars

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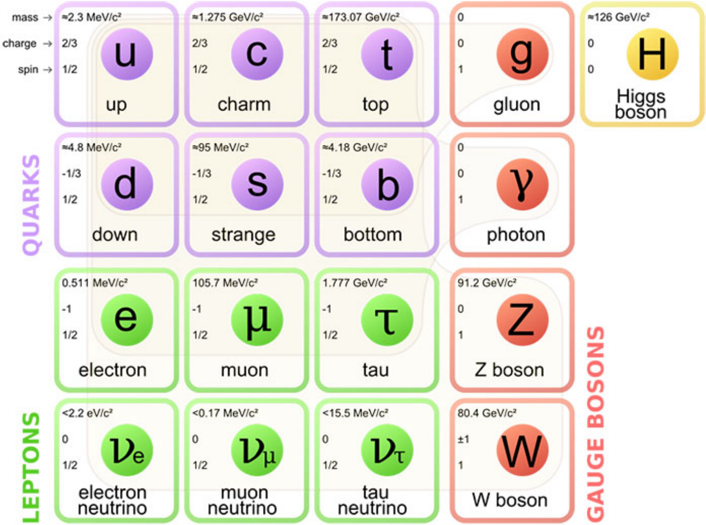
Outline

1. Introduction to Particle Physics and QCD
2. Introduction to Neutron Star Physics
3. What is the QCD perspective on Neutron Stars?
4. The Dense QCD Trace Anomaly in Twin Stars
5. Summary and Outlook

1. Current Paradigm of Particle Physics



The Standard Model of Particle Physics

1

$$-\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc}\partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \frac{1}{2}ig_s^2(\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g_\mu^c$$

2

$$- \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - igs_w[\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - gMW_\mu^+ W_\mu^- H - \frac{1}{2}g\frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig[W_\mu^+(\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^-(\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g[W_\mu^+(H\partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^-(H\partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z_\mu^0(H\partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig\frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + igs_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig\frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + igs_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+\phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w}(2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^-$$

3

$$- \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda - \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + igs_w A_\mu\left[-(\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)\right] + \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1+\gamma^5)e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1+\gamma^5)C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1+\gamma^5)u_j^\lambda)] +$$

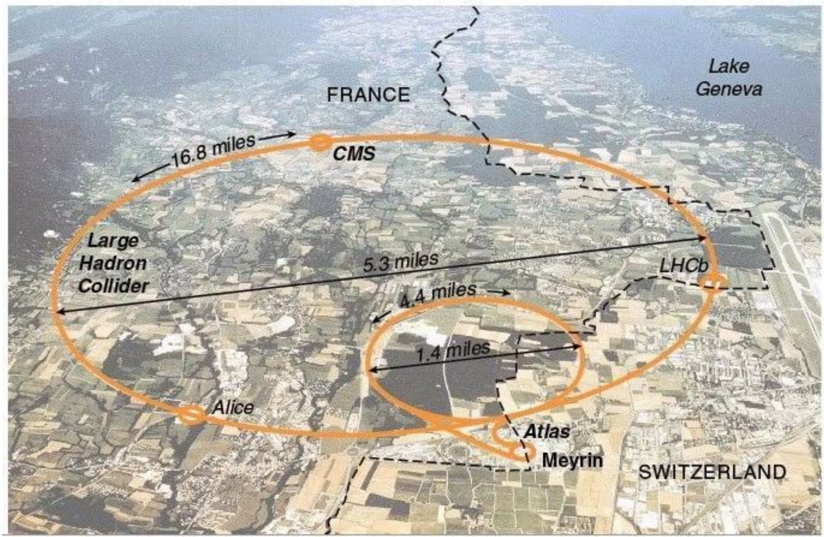
4

$$\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) + m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1+\gamma^5)d_j^\kappa] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1-\gamma^5)u_j^\kappa] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda \gamma^5 d_j^\lambda) +$$

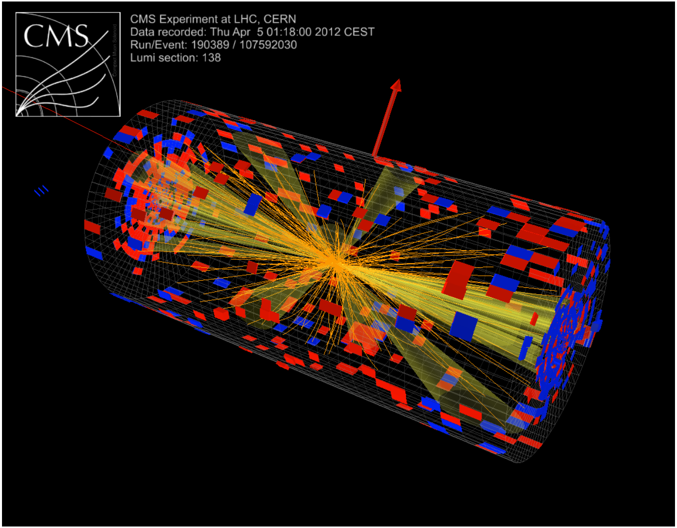
5

$$\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W_\mu^+(\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+(\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^-(\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + igs_w W_\mu^-(\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu(\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

Collider Experiments: LHC at CERN



Collider Experiments: LHC at CERN



[CMS webpage, 2023]

Brief History of Nuclear-Particle Physics

Theory		Experiment	
		1897	Thomson's discovery of electron e^-.
1918	Weyl's first gauge concept.		
		1919	Rutherford's discovery of proton p.
		1922	Confirmation that photon is elementary (Compton).
1928	Dirac's prediction of anti-particles.		
1929	Weyl's gauge theory of electromagnetism.		
		1932	Anderson discovers positron. Evidence for neutron (Chadwick).
1934	Fermi's theory of weak interactions.		
1935	Yukawa's prediction of the meson.		
		1947	Discovery of π-meson and μ-lepton.
1954	Yang–Mills/Utiyama gauge field theory.		
1956	Lee and Yang predict non-conservation of parity in weak interactions.	1956	Detection of neutrino (Reines and Cowan). Wu *et al.* discover parity violation.
1958	V–A theory of weak interactions.		
1961	Weak neutral-currents predicted (Glashow).		
1964	Higgs mechanism. Quarks and strong force (Gell-Mann; Zweig). Coloured quarks and gluons (Greenberg; Han and Nambu).		
1967	Electroweak unification (Weinberg; Salam; Glashow).		
1971	Renormalizability of gauge theories with Spontaneous Symmetry Breaking ('t Hooft).		
1973	Quantum Chromodynamics Lagrangian (Fritzsch, Gell-Mann and Leutwyler).	1973	Weak neutral-currents detected.
		1974	Evidence of c-quark from the J/ψ resonance.
		1975	Evidence of τ-lepton.
		1977	Evidence of b-quark from the Υ resonance.
		1979	Evidence for the gluon in $e^+e^- \to 3$ jet.
		1983	$W^\pm$, Z bosons discovered.
		1994	Evidence for the t-quark.

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THE EUROPEAN PHYSICAL JOURNAL C

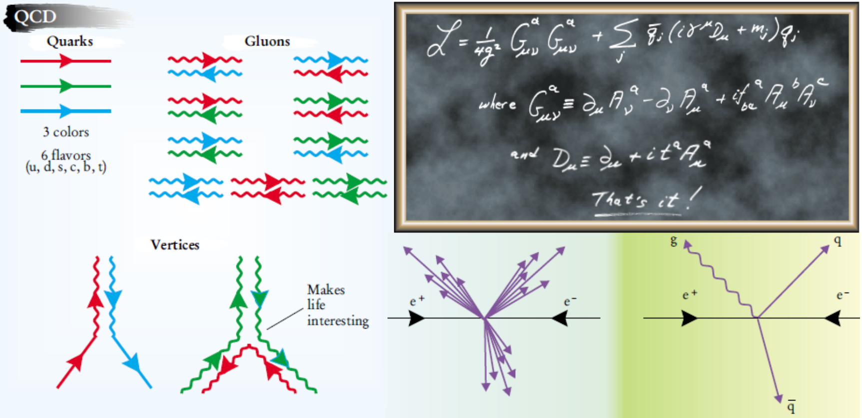
Review

50 Years of quantum chromodynamics

Introduction and Review

Franz Gross[1,2,a], Eberhard Klempt[3,b], Stanley J. Brodsky[4], Andrzej J. Buras[5], Volker D. Burkert[1], Gudrun Heinrich[6], Karl Jakobs[7], Curtis A. Meyer[8], Kostas Orginos[1,2], Michael Strickland[9], Johanna Stachel[10], Giulia Zanderighi[11,12], Nora Brambilla[5,12,13], Peter Braun-Munzinger[10,14], Daniel Britzger[11], Simon Capstick[15], Tom Cohen[16], Volker Crede[15], Martha Constantinou[17], Christine Davies[18], Luigi Del Debbio[19], Achim Denig[20], Carleton DeTar[21], Alexandre Deur[1], Yuri Dokshitzer[22,23], Hans Günter Dosch[10], Jozef Dudek[1,2], Monica Dunford[24], Evgeny Epelbaum[25], Miguel A. Escobedo[26], Harald Fritzsch[27], Kenji Fukushima[28], Paolo Gambino[11,29], Dag Gillberg[30,31], Steven Gottlieb[32], Per Grafstrom[33,34], Massimiliano Grazzini[35], Boris Grube[1], Alexey Guskov[36], Toru Iijima[37], Xiangdong Ji[16], Frithjof Karsch[38], Stefan Kluth[11], John B. Kogut[39,40], Frank Krauss[41], Shunzo Kumano[42,43], Derek Leinweber[44], Heinrich Leutwyler[45], Hai-Bo Li[46,47], Yang Li[48], Bogdan Malaescu[49], Chiara Mariotti[50], Pieter Maris[51], Simone Marzani[52], Wally Melnitchouk[1],

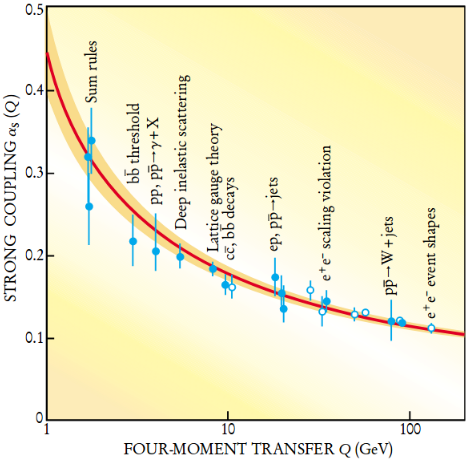
Quantum Chromodynamics (QCD)



[Wilczek, 2000]

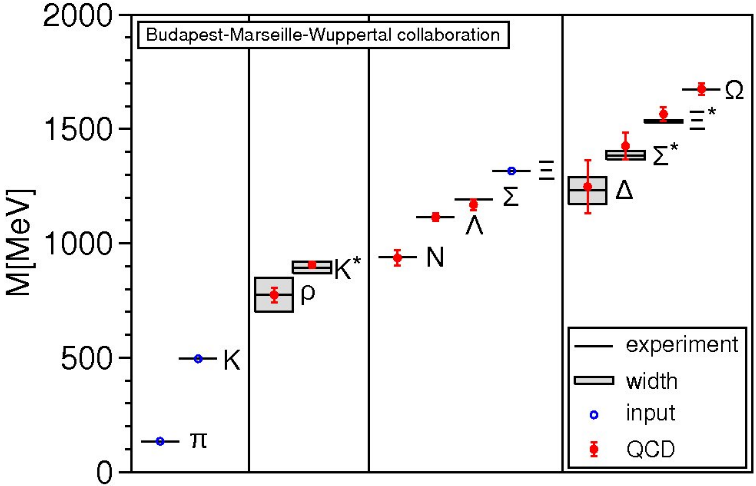
*Notable Property: Asymptotic Freedom

Nobel Prize of Physics 2004 - Wilczek/Gross and Politzer



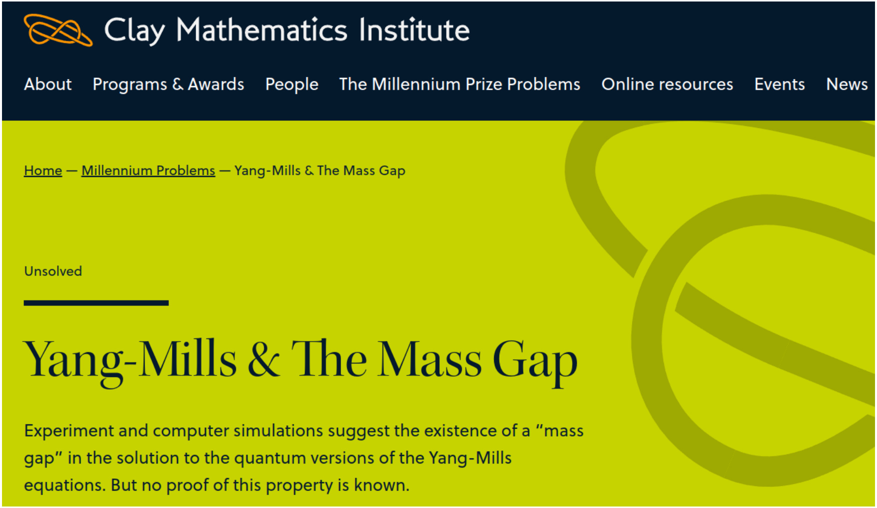
[Wilczek, Phys. Today (2000)]

*Notable Property (?): Color Confinement



Light and Heavy Hadron Masses [Durr et al., Science (2008)]

Hardest Millenium Unsolved Problem:



[As of 25/02/2025]

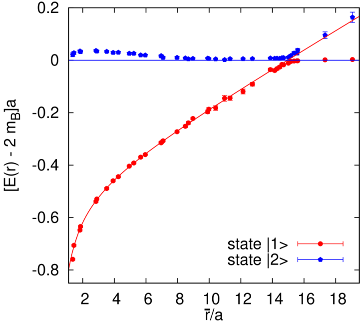
Non-perturbative QCD Vacuum Structure

We show QCD animations obtained by [D. Leinweber, 2003-2004]:

- The Euclidean Action Density (or Energy Density)

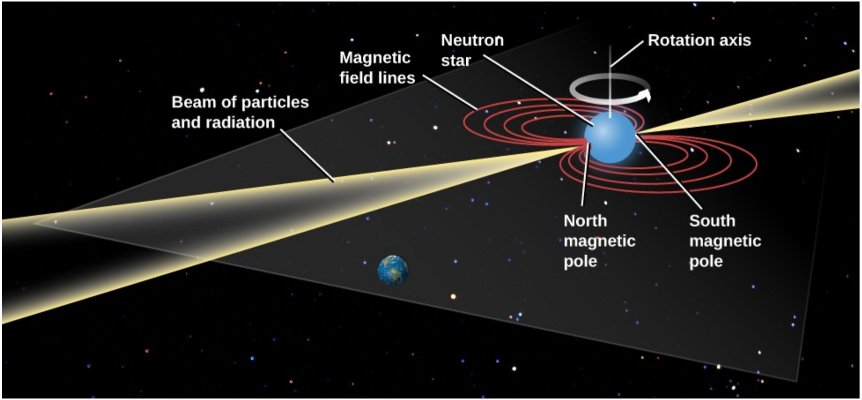
$$S_E(\vec{x},t) = \frac{1}{2}F^{ab}_{\mu\nu}(\vec{x},t)F^{ba}_{\mu\nu}(\vec{x},t) = \mathrm{Tr}\left(\vec{E}^2(\vec{x},t) + \vec{B}^2(\vec{x},t)\right)$$

- Flux tubes in QCD ground-state vacuum fields:



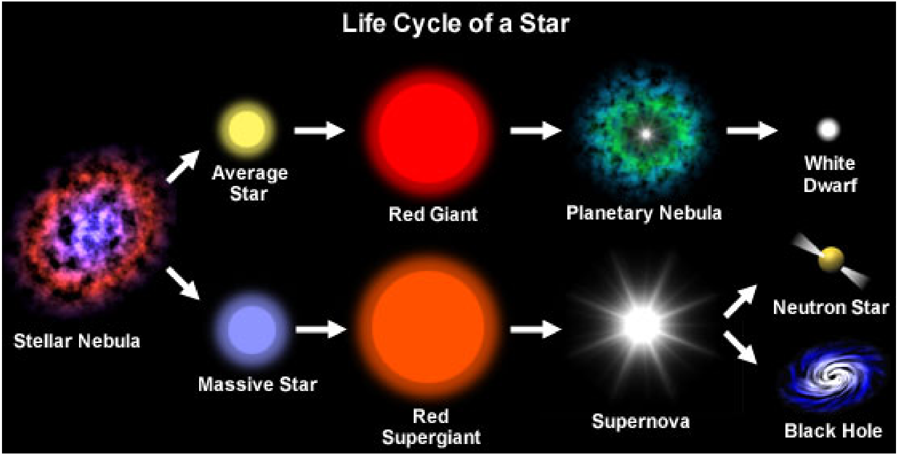
$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r \qquad \text{(Cornell Potential).}$$

2. Pulsating Source of Radiation → **Pulsar**



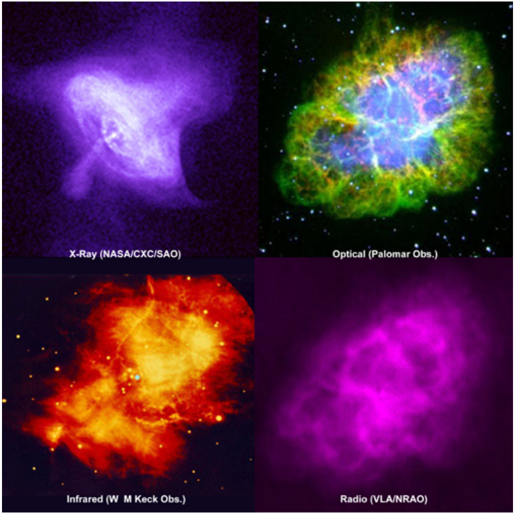
[https://cnx.org/contents/v-2IbQIC@10/Pulsars-and-the-Discovery-of-Neutron-Stars]

Stages of stellar evolution (very simplified plot!)



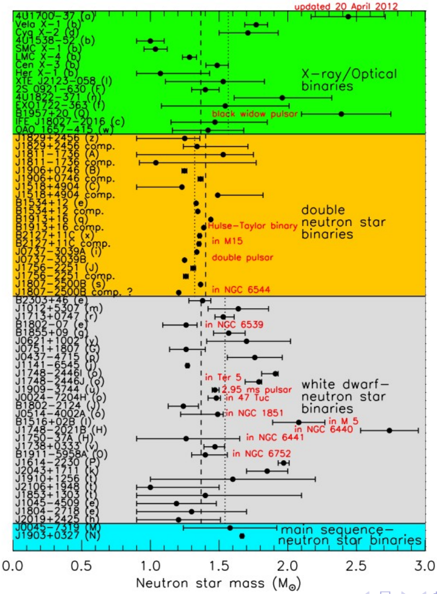
Nuclear-matter formation through gravitational-collapse processes

Example: The famous Crab nebula

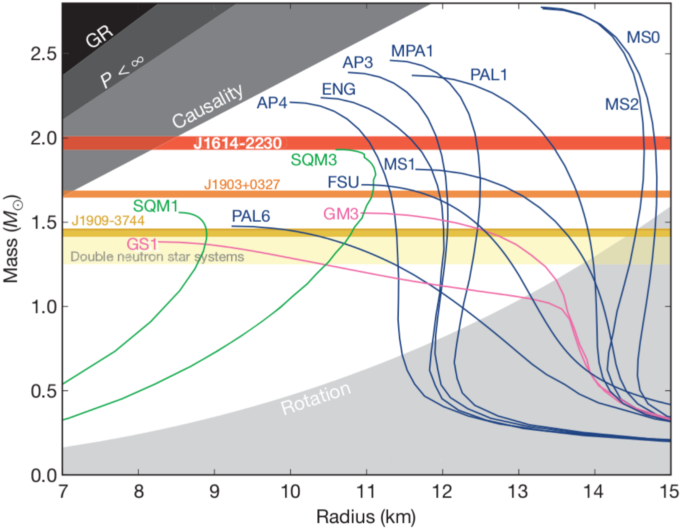


This nebula contains a fast rotating neutron star

Pulsar Mass Observations [Lattimer, 2012]

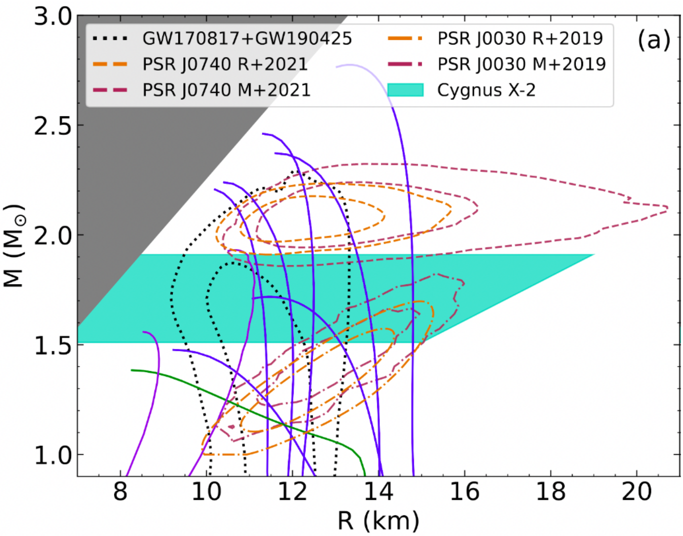


Maximal mass NS constraint



Mass limit of $\sim\ 2M_{\odot}$ for NS in binary systems [Demorest, 2010].

Radii data by NICER for canonical and maximal NS



Bands of data from NICER (Neutron Star Interior Composition Explorer)

Stellar Structure: The TOV Equations

- Tolman (1934) and Oppenheimer with Volkov (1939) derived the equations for hydrostatic equilibrium in relativistic stars in order to obtain their, in principle, observable **masses** and **radii**.
- These equations are

$$\frac{dP}{dr} = -\frac{G\mathscr{M}(r)\epsilon(r)}{r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{\mathscr{M}(r)}\right]\left[1-\frac{2G\mathscr{M}(r)}{r}\right]^{-1},$$

$$\frac{d\mathscr{M}}{dr} = 4\pi r^2\epsilon(r),$$

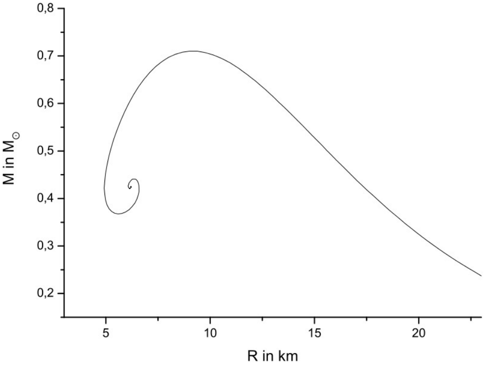
with boundary conditions + physical conditions

$$P(r=0)=P_0, \quad \mathscr{M}(r=0)=0, \quad P(r=R)=0, \quad \mathscr{M}(R)=M.$$

- To be solved consistently, one needs the microphysics input called Equation of State (EoS): $P = P(\epsilon)$ or $\epsilon = \epsilon(P)$.

The Oppenheimer-Volkov limit (1939)

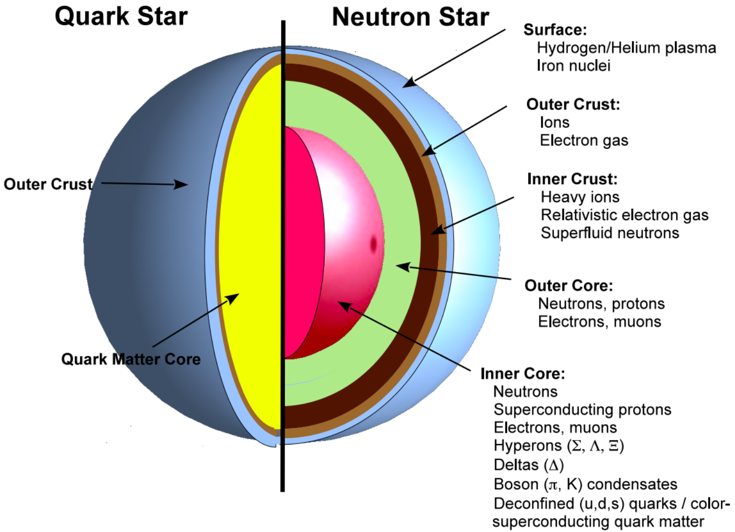
- They modeled NS as a gas of relativistic degenerate neutrons in hydrostatic equilibrium with gravity, thus obtaining



(Sagert, Hempel, Greiner, JSB 2006)

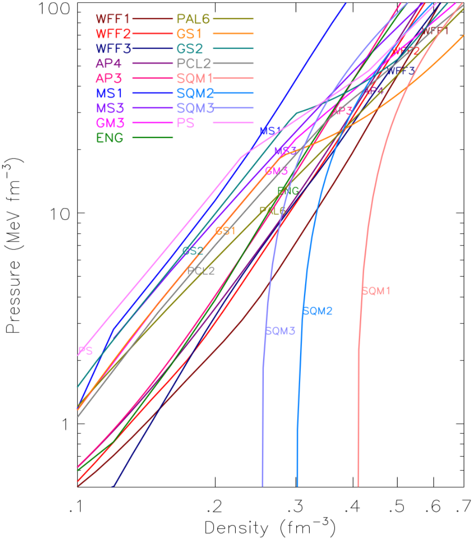
- Then, **interactions + new phases are important !**

Neutron-star interiors with exotic phases



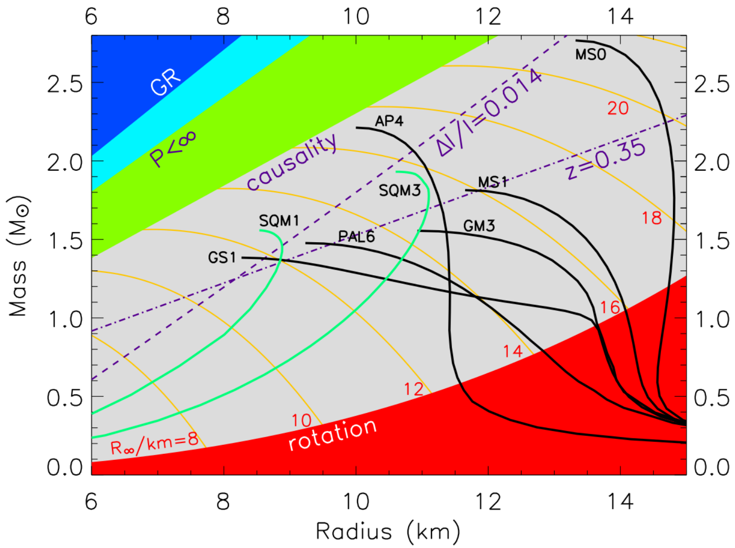
[Weber, 2012]

Several existing EoS in the literature



[J. Lattimer, 2005]

Several different behaviors in the MR diagram



[J. Lattimer, 2005]

What about their dynamical stability?

By Newton's 2nd law one has

$$m\frac{d^2}{dt^2}x = -\frac{dV}{dx}$$

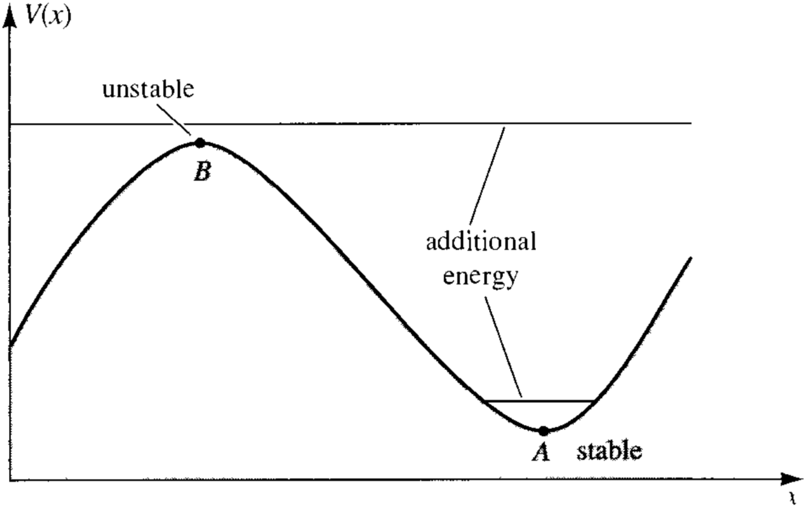
which becomes the following if assuming small perturbations $\xi(t)$ around a position of mechanical equilibrium $x_{A,B}$ (a constant), i.e. $x(t) = x_{A,B} + \xi(t) + \mathcal{O}(\xi^2)$:

$$m\frac{d^2}{dt^2}\xi = -\left(\frac{\partial^2 V}{\partial x^2}\right)_{x_{A,B}} \xi.$$

Now, assuming a harmonic perturbation one would use as a reasonable ansatz $\xi(t) \propto \exp(\pm i\omega_{A,B} t)$, thus producing

$$\omega^2_{A,B} \equiv \frac{1}{m}\left(\frac{\partial^2 V}{\partial x^2}\right)_{x_{A,B}}$$

What about their dynamical stability?



[J. Hartle's 'Gravity', 2003]

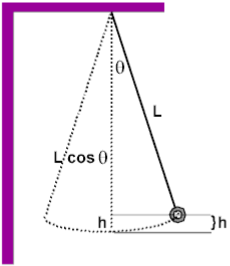
Example: The Famous 2D Pendulum

For this well-known oscillating problem, one has a potential energy of the form

$$V(\theta) = mgL\cos\theta,$$

thus giving

$$\omega^2 < 0, \text{ for } \theta \in [\pi/2, 3\pi/2].$$



Relativistic Stellar Stability: General

Defining $\Delta r/r \equiv \xi$ and ΔP as the independent variables for the pulsation problem, one gets the coupled differential equations [Gondek *et al.*, 1997]:

$$\frac{d\xi}{dr} = -\frac{1}{r}\left(3\xi + \frac{\Delta P}{\Gamma P}\right) - \frac{dP}{dr}\frac{\xi}{(P+\epsilon)} ,$$

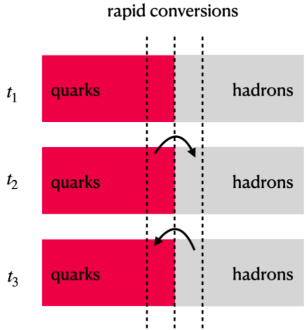
and

$$\frac{d\Delta P}{dr} = \xi\left\{\omega^2 e^{\lambda-\nu}(P+\epsilon)r - 4\frac{dP}{dr}\right\} + \xi\left\{\left(\frac{dP}{dr}\right)^2\frac{r}{(P+\epsilon)} - 8\pi e^{\lambda}(P+\epsilon)Pr\right\} + \Delta P\left\{\frac{dP}{dr}\frac{1}{P+\epsilon} - 4\pi(P+\epsilon)re^{\lambda}\right\} ,$$

where ω is the oscillation frequency.

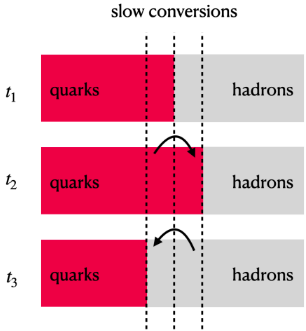
Relativistic Stellar Stability: 1st-order Phase Transitions in Hybrid Neutron Stars

$\tau_{reactions} \ll \omega_0^{-1} \sim 1$ ms



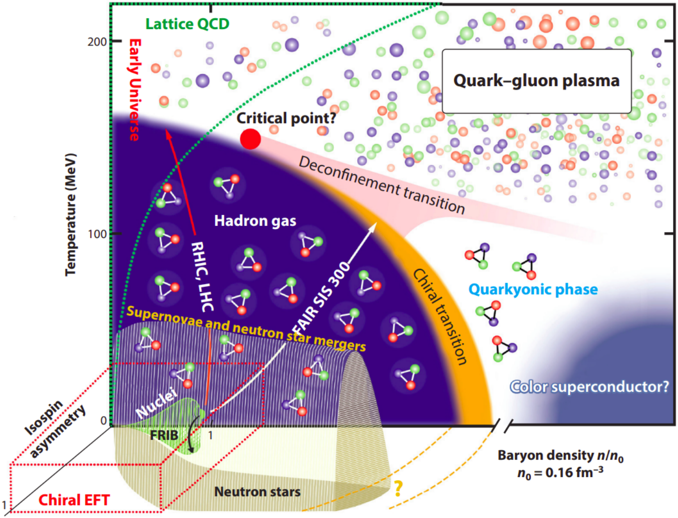
$$\begin{cases} \Delta p^+ = \Delta p^- \\ \left[\xi - \frac{\Delta p}{r p_0'}\right]^+ = \left[\xi - \frac{\Delta p}{r p_0'}\right]^- \end{cases}$$

$\tau_{reactions} \gg \omega_0^{-1} \sim 1$ ms



$$\begin{cases} \Delta p^+ = \Delta p^- \\ \xi^+ = \xi^- \end{cases}$$

3. Phase diagram (cartoon) of QCD



[Drischler et al., 2021].

Thermal and dense QCD:
Simple prescription but a challenging calculation

- The total pressure of a QCD can be obtained from

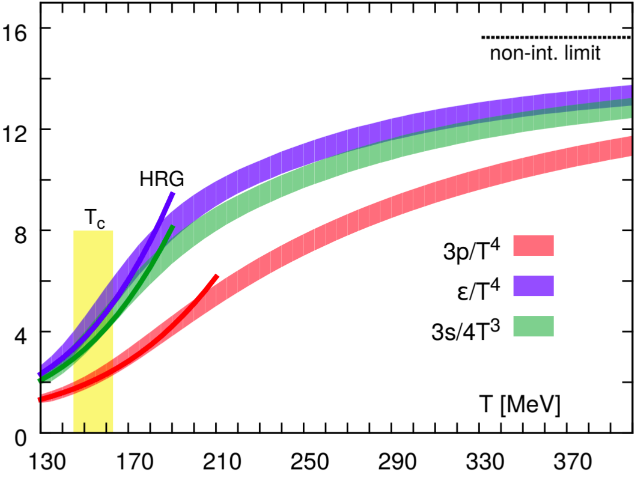
$$P(T,\{\mu_i\}) = T \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x \int_0^{1/T} d\tau \mathcal{L}^{\mathrm{E}}_{\mathrm{QCD}}}.$$

- For $T \neq 0$ and $\mu \lesssim T$: Lattice-gauge-field theory methods **apply**.

- For $\mu \gtrsim T$: **Unfeasible** due to the fermionic sign problem.
  In general, this is an example of the NP $\neq$ P conjecture proposed as a Millenium Problem still lacking a proof [arXiv: 0408370, 2007.05436].

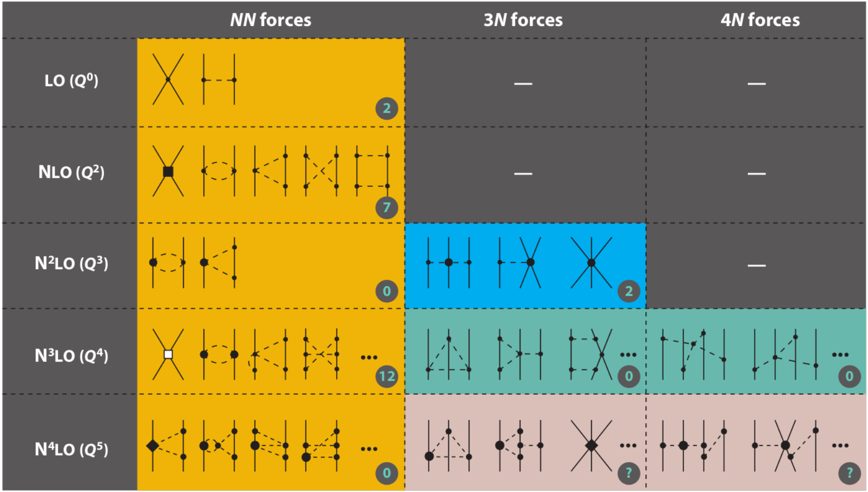
- Perturbative control at low densities (chiral effective field theory) and at ultra-high densities (perturbative QCD through $\alpha_s(\mu) \sim 1/\log(\mu^2)$), both in the **cold limit**.

The lattice QCD equation of state at finite ‘T’



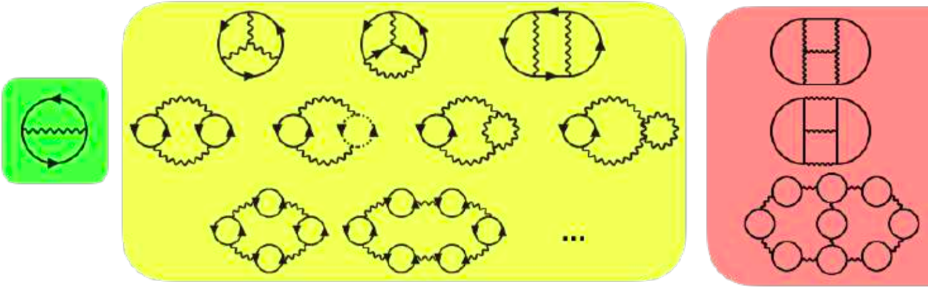
[A. Bazavov et al., 2014]

Chiral Effective (Perturbation) Theory



Hierarchy of chiral nuclear interactions up to fifth order
in the chiral expansion [C. Drischler et al., 2010].

Cold and dense perturbative QCD (pQCD)



$$P(\mu_B)/P_{\text{free}} \sim 1 + \underbrace{c_1 g^2}_{NLO} + \underbrace{c_2 g^4 + c_2' g^4 \log g}_{NNLO} + \underbrace{c_3' g^6 \log^2 g + c_3'' g^6 \log g + \dots}_{N^3LO}$$

Ex.: Constraining the neutron star equation of state

nature physics

LETTERS

https://doi.org/10.1038/s41567-020-0914-9

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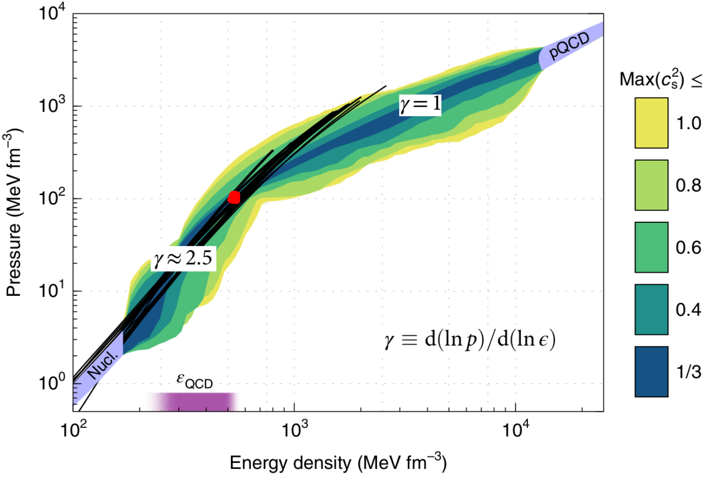
Evidence for quark-matter cores in massive neutron stars

Eemeli Annala[1], Tyler Gorda[2]✉, Aleksi Kurkela[3,4]✉, Joonas Nättilä[5,6,7] and Aleksi Vuorinen[1]✉

[Nature Phys. 16 (2020) 9, 907-910]

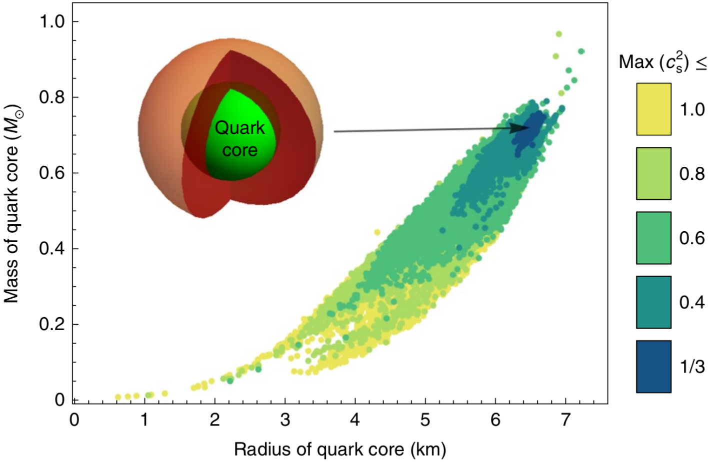
Ex.: Constraining the neutron star equation of state



Equations of state for QCD matter

[Annala *et al.*, Nature Phys. 2020]

Ex.: Constraining the neutron star equation of state



Quark-matter core masses in NSs

[Annala *et al.*, Nature Phys. 2020]

4. Some Recent Related Work

PHYSICAL REVIEW D **110,** 114014 (2024)

How the QCD trace anomaly behaves at the core of twin stars?

José C. Jiménez,[1,2] Lucas Lazzari,[3] and Victor P. Gonçalves[3,4]

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[2]*Universidad Tecnológica del Perú, Arequipa - Perú*

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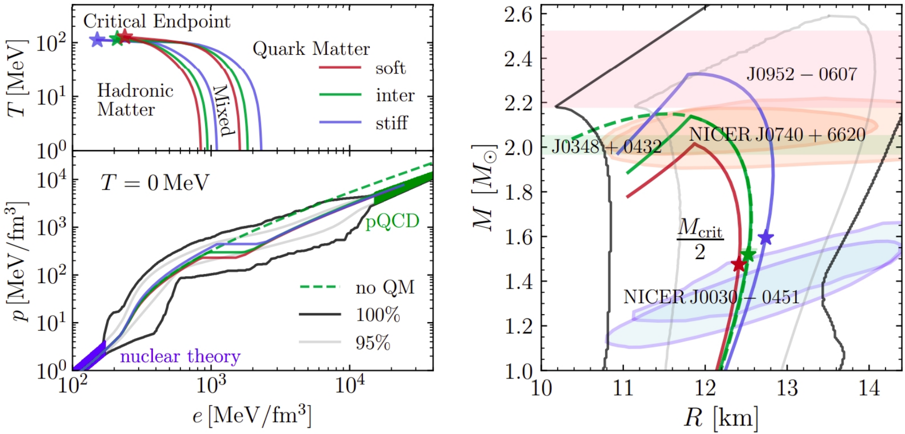
We investigate the behavior of the dense and cold (normalized) quantum chromodynamics (QCD) trace anomaly, Δ, in the interior of twin neutron stars (obtained from several sets of equations of state in agreement with modern compact-star and multimessenger data) satisfying static and dynamic stability conditions. We scan the formed twin-star parameter space in order to look for effects caused by the presence of a strong first-order phase transition connecting hadron and quark phases by means of a Maxwell construction. We found robustly that Δ suffers an abrupt decrease around the transition point, even reaching large negative values ($\Delta \simeq -0.35$), in marked contrast to current studies pointing out a

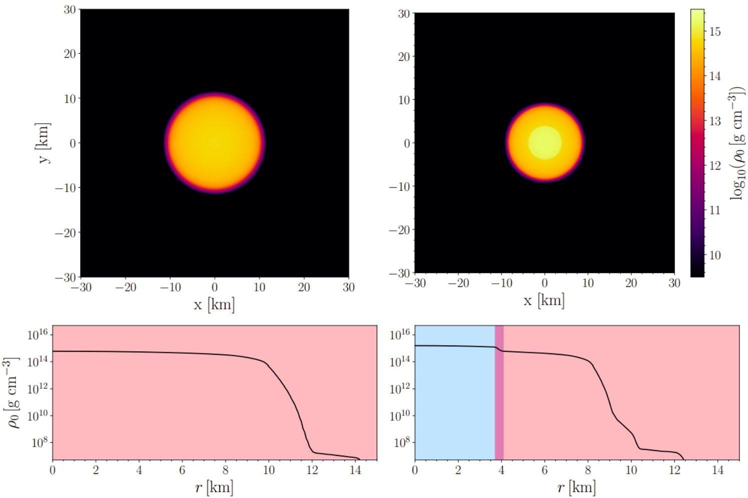
[JCJ et al., 2024]

Motivation



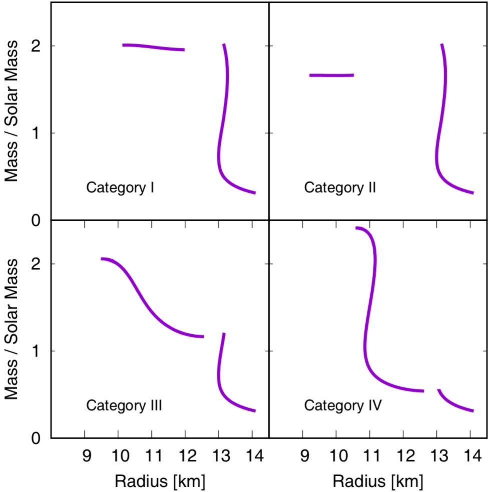
Phase boundaries and EoS (left)
and corresponding *M*–*R* diagram (right) [Ecker et al., 2402.11013]

Motivation



Dynamical general-relativistic twin-star formation
[Naseri *et al.*, 2406.15544]

Twin-star matter essentials



Categories of twin stars according to their *M*'s
[J-E Christian *et al.*, Eur. Phys. J. A (2018) 54:28]

Trace anomaly in dense matter

- QCD trace anomaly as measure of breaking conformal invariance:

$$\eta_{\mu\nu} T^{\mu\nu}_{\mathrm{QCD}} \equiv T^{\mu}_{\mu} = \frac{\beta_{\mathrm{QCD}}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + (1+\gamma_m) \sum_f m_f \overline{q}_f q_f \,.$$

- Thermal/dense case:

$$\left\langle T^{\mu}_{\mu} \right\rangle_{\mu_B, T} = \epsilon - 3P.$$

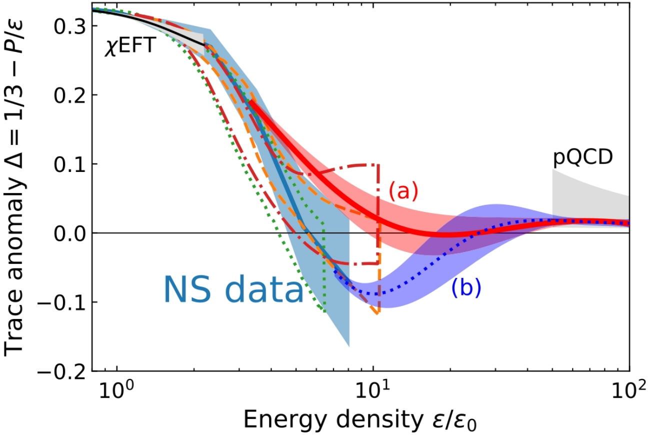
- Normalized thermal/dense case:

$$\Delta \equiv \frac{\langle T^{\mu}_{\mu} \rangle_{\mu_B, T}}{3\epsilon} = \frac{1}{3} - \frac{P}{\epsilon}.$$

- Causality ($P = \epsilon$, i.e. $c_s^2 = 1$) and non-relativistic ($P \ll \epsilon$) limits

$$-\frac{2}{3}(\approx -0.667) \leq \Delta < \frac{1}{3}(\approx 0.333).$$

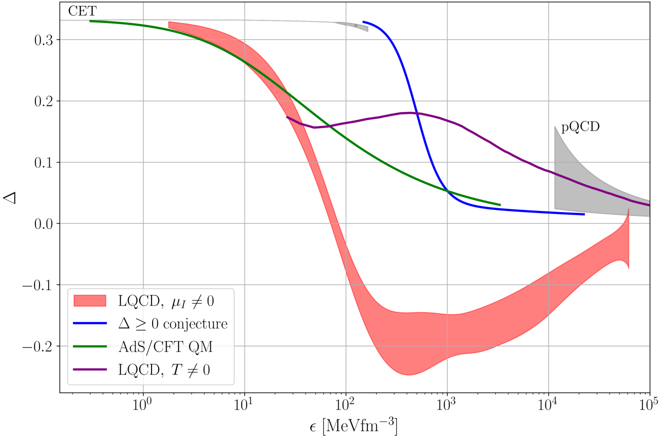
Trace anomaly in neutron-star interiors



Trace anomaly behavior with different NS data

[Y. Fujimoto *et al.*, PRL 129, 252702 (2022)]

In-medium Trace Anomaly in QCD Matter



Behavior of Δ for different kinds of extreme matter

[J. C. J. *et al.*, 2408.11614]

Twin-star Matter and Seidov's Criterium

- Constant-speed-of-sound parametrization for the equation of state

$$\epsilon(P) = \begin{cases} \epsilon_{\mathrm{H}}(P) & P < P_t \,, \\ \epsilon_{\mathrm{H}}(P_t) + \Delta\epsilon + s^{-1}(P - P_t) & P > P_t \,. \end{cases}$$

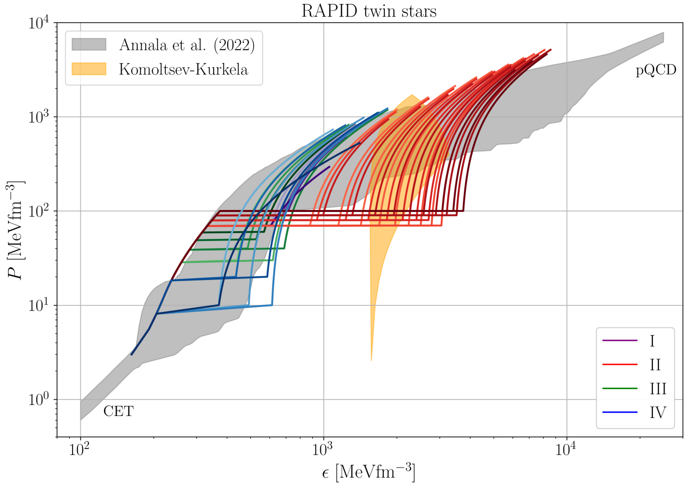
- Seidov's criterium to ensure the twin-star branch in the MR diagram

$$\Delta\epsilon \geq \Delta\epsilon_{\mathrm{crit}} \equiv \frac{1}{2}\epsilon_t + \frac{3}{2}P_t .$$

- Particular set of parameters (in units of $\mathrm{MeV\,fm^{-3}}$) used

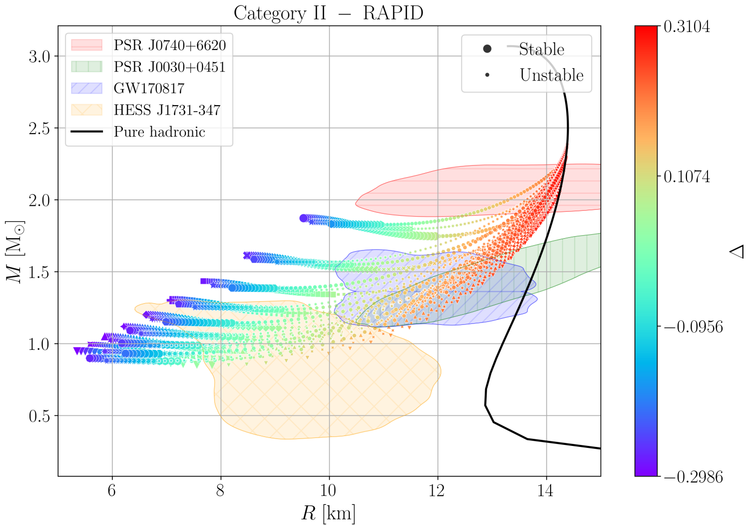
Category	$\epsilon_H^{\mathrm{max}} = \epsilon_t$	$\epsilon_Q^{\mathrm{min}}$	P_t	$\Delta\epsilon$	c_s^2
I	333.08	607.34	70	274	1
II	333.08	878.88	70	545	1
III	263.73	441.62	30	178	1
IV	212.91	370.85	10	157	1

Studied Twin-Star Equations of State

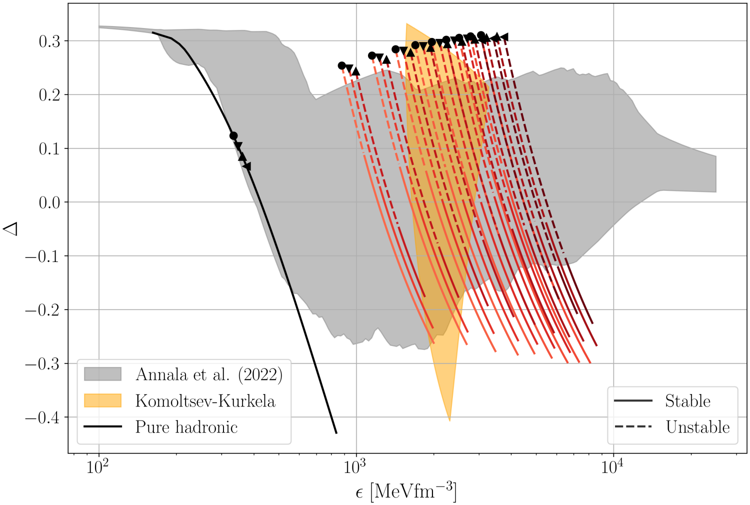


Family of EoSs for Category I-IV stable twin stars with rapid conversions.

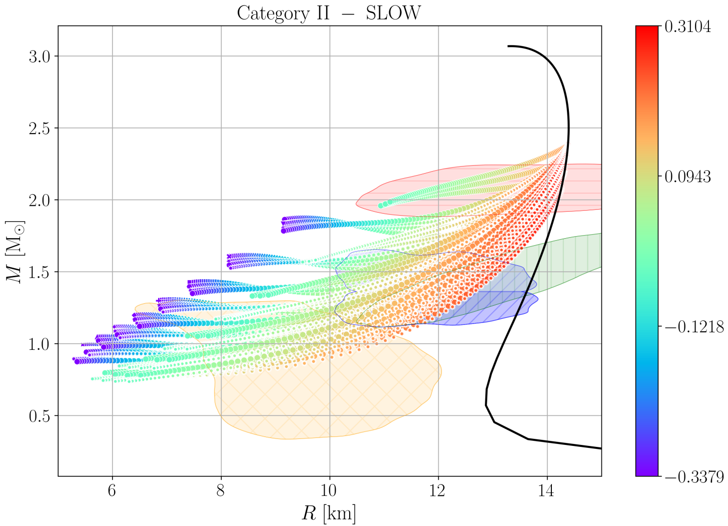
M–R for *rapid* Category II twin stars



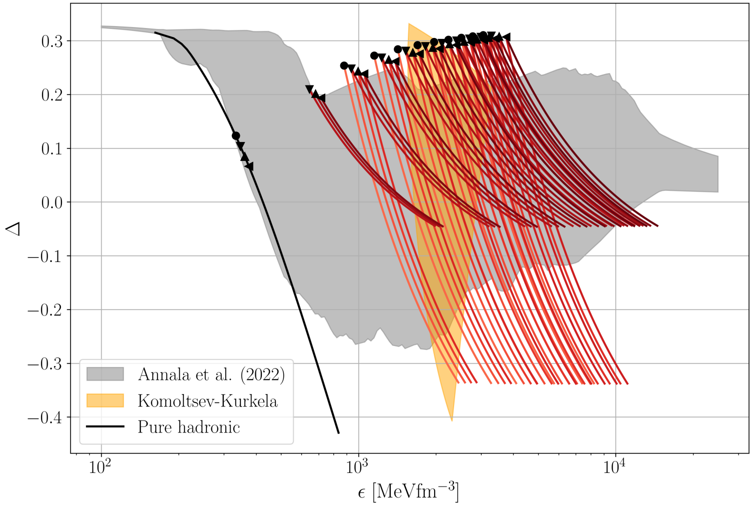
△ for *rapid* Category II twin stars



M–R for *slow* Category II twin stars



Δ for *slow* Category II twin stars



Some insights for dense QCD

- Conjecture of $\Delta > 0$ (Fujimoto et al., 2022) through

$$\frac{\epsilon - 3P}{P_{\rm ideal}} = \mu_B \frac{dN_{\rm eff}}{d\mu_B} > 0,$$

where $N_{\rm eff} \equiv P/P_{\rm ideal}$ and $P_{\rm ideal} \equiv N_c N_f \frac{\mu_B^4}{12\pi^2}$.

- In our case, a finite latent heat, Q, is present:

$$Q = \mu_c \Delta n_B = \left\langle T^\mu_\mu(\mu_B^+ \to \mu_c)\right\rangle_{\rm Q} - \left\langle T^\mu_\mu(\mu_B^- \to \mu_c)\right\rangle_{\rm H},$$

or equivalently

$$\frac{Q}{\mu_c^4} = \mu_c \left[\left(\frac{dN^Q_{\rm eff}}{d\mu_B^+}\right) - \left(\frac{dN^H_{\rm eff}}{d\mu_B^-}\right)\right]_{\mu_B^\pm \to \mu_c}.$$

Summary and Outlook

- The **standard model** of particle physics is better to be understood as an **effective theory**.
- Color confinement and (hadron) mass generation from QCD (from an **analytic viewpoint**) are the **hardest** and **relevant** questions of modern theoretical physics.
- Multimessenger astrophysics is expected to give insights into the **building** and **constraining** of the equation of state for QCD matter.
- The modern paradigm in QCD is that even if we don't have full answers, **insights** should be gained in every possible way.

But don't forget ...