Regular black holes in modified third-order gravitational theories in curvature

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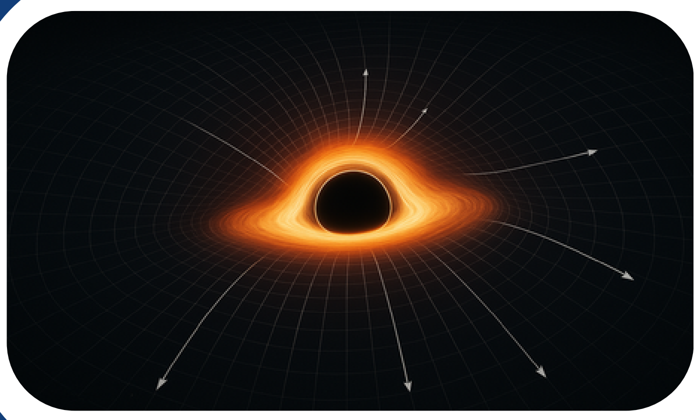
Introduction and Motivation



Singularities are regions of spacetime where the metric becomes ill-defined. In solutions such as the Schwarzschild or Gauss–Bonnet black holes, the presence of these singularities is evident near the origin. Several approaches have been proposed to address this issue, including the introduction of scalar fields, the incorporation of dark matter, or the inclusion of higher-order curvature terms. In this work, quasi-topological Lagrangian densities are constructed up to third order in curvature, and the minisuperspace method is applied to derive the equations of motion. Furthermore, the conditions under which the additional terms remove the singularity are established.

Quasi-topological gravities

For the static and spherically symmetric metric (SSS):

$$ds^2_{N,f} = -N(r)^2 f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{(D-2)}$$

Lagrangian densities Z(i) can be constructed and added to the Ricci scalar into the Hilbert–Einstein action.

$$I = \frac{1}{16\pi G}\int d^5x\sqrt{-g}[R + \sum_{i=2}^{n}\alpha(i)Z(i)]$$

It is convenient to employ quasi-topological Lagrangian densities, since the solutions they yield do not contain fourth-order derivatives but only up to second order, and may even admit algebraic solutions.

For a Lagrangian density to belong to the quasi-topological family, it must satisfy the following condition:

$$\frac{\delta S_f}{\delta f} = 0\,, \quad \forall\ f(r)$$

In this context, the Schwarzschild solution corresponds to a first-order quasi-topological term, the Gauss–Bonnet one to a second-order term, and in this work, third-order terms will be studied.

Methods

The method used to obtain the equations of motion is the **Minisuperspace** approach, which consists of reducing the action to an effective Lagrangian by substituting a suitably symmetric metric, and then applying the Euler–Lagrange equations.

$$I = \Omega_{(D-2)}\int dt \int dr\mathcal{L}_{N,f}$$

$$\frac{\partial\mathcal{L}_{N,f}}{\partial N} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial\mathcal{L}_{N,f}}{\partial N'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial\mathcal{L}_{N,f}}{\partial N''} = 0$$

$$\frac{\partial\mathcal{L}_{N,f}}{\partial f} - \frac{\mathrm{d}}{\mathrm{d}r}\frac{\partial\mathcal{L}_{N,f}}{\partial f'} + \frac{\mathrm{d}^2}{\mathrm{d}r^2}\frac{\partial\mathcal{L}_{N,f}}{\partial f''} = 0$$

The third-order invariants of curvature are:

$$Z_1 = R_{a\ b}^{\ c\ d}R_{c\ d}^{\ e\ f}R_{e\ f}^{\ a\ b} \quad , \quad Z_2 = R_{ab}^{\ \ cd}R_{cd}^{\ \ ef}R_{ef}^{\ \ ab} \quad , \quad Z_3 = R_{abcd}R^{abc}{}_{e}R^{de}$$

$$Z_4 = R_{abcd}R^{abcd}R \quad , \quad Z_5 = R_{abcd}R^{ac}R^{bd} \quad , \quad Z_6 = R_a^{\ b}R_b^{\ c}R_c^{\ a}$$

$$Z_7 = R_a^{\ b}R_b^{\ a}R \quad , \quad Z_8 = R^3 \quad , \quad Z_9 = \nabla_a R_{bc}\nabla^a R^{bc} \quad , \quad Z_{10} = \nabla_a R\nabla^a R$$

These terms make up the third-order Lagrangian density where c represents the coefficients of each invariant:

$$Z(3) = c_1Z_1 + c_2Z_2 + c_3Z_3 + c_4Z_4 + c_5Z_5 + c_6Z_6 + c_7Z_7 + c_8Z_8 + c_9Z_9 + c_{10}Z_{10}$$

Regular Black Holes

For a Lagrangian density to be of the quasi-topological type, the following condition must be satisfied:

$$c_4 = \frac{9c_1 - 156c_2 - 56c_3}{216} \quad , \quad c_5 = -\frac{9c_1 + 168c_2 + 52c_3}{27} \quad , \quad c_6 = \frac{36c_1 - 408c_2 - 134c_3}{81}$$

$$c_7 = \frac{-9c_1 + 372c_2 + 92c_3}{54} \quad , \quad c_8 = \frac{9c_1 - 588c_2 - 128c_3}{648} \quad , \quad c_9 = c_{10} = 0$$

Thus, the complete action up to third order in curvature can be written as:

$$I = \frac{1}{16\pi G}\int d^5x\sqrt{-g}[R + \alpha Z(2) + \alpha^2 Z(3)]$$

From the Euler–Lagrange equations with respect to N, the following algebraic equation is obtained:

$$\alpha^2 f^3 + (-r^2\alpha - 3\alpha^2)f^2 + (3r^4 + 2r^2\alpha + 3\alpha^2)f - (3r^4 + \alpha^2 - r^2\mu) = 0$$

As can be seen in the graphs, the singularity at r=0 disappears in the third-order curvature black hole.

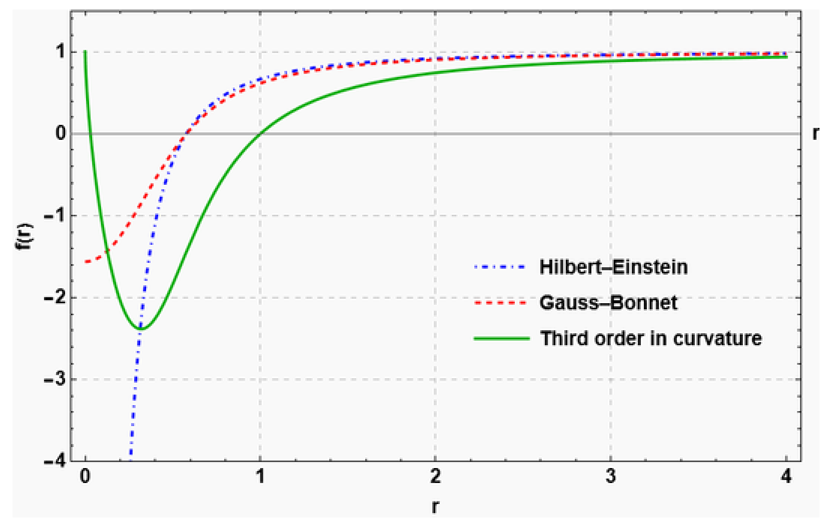


Figure 1. Comparative graph between black hole solutions with $\alpha = 0.03$

As can be seen in Fig. 2, f(r) does not exhibit singularity for different values of mu. Furthermore, the Kretschmann invariant is finite for all values of r.

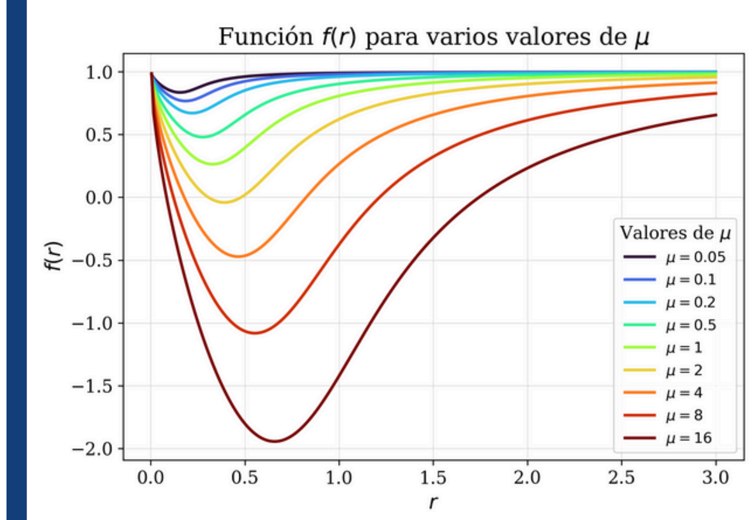


Figure 2. Third-order black hole solution in curvature for different values of mu

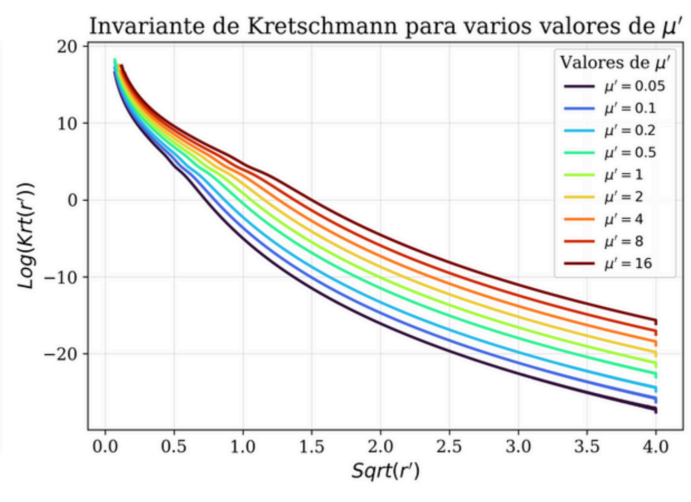


Figure 3. Kretschmann invariant for different values of mu

Conclusiones

- Working with quasi-topological Lagrangian densities greatly simplifies the calculations required to obtain black hole solutions.
- Regular black hole solutions can be obtained by including curvature terms of order higher than n=3.
- There exist multiple regular black hole solutions, whose form depends on the type of coupling employed.
- Graphically, it can be observed that there is no singularity at r=0; however, to ensure its absence with certainty, the Hawking–Penrose theorems on geodesic incompleteness should be applied.

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