Robust Quantum Control via the dark state: Adiabatic population engineering in a three-level Λ system

Rosario Aldava Asencios [1] Pedro Rojas Fernández [1] Teófilo Vargas Auccalla[1]
Ricardo Quispe-Mendizábal[1,2]

[1]GFT & QuantumQuipu, Universidad Nacional Mayor de San Marcos [2]Universidad de Santiago de Chile

Abstract

Achieving high-fidelity quantum state engineering is pivotal for quantum technology applications. This work investigates the Stimulated Raman Adiabatic Passage (STIRAP) protocol in a three-level Λ-system to demonstrate robust and near-perfect population transfer ($|1\rangle \to |2\rangle$). The system evolution is rigorously modeled using the **Lindblad Master Equation** to account for realistic open quantum dynamics, specifically spontaneous decay from the intermediate state $|3\rangle$. We show that the **adiabatic evolution** through the Dark State, which is orthogonal to the decaying state $|3\rangle$, guarantees immunity to this dissipation, achieving fidelities > 0.99. Furthermore, we analyze the protocol's resilience, mapping the fidelity landscape in the parameter space and confirming STIRAP's intrinsic robustness against control amplitude variations. This study demonstrates the efficacy and practical viability of STIRAP as a robust method for coherent population transfer in realistic noisy quantum devices.

Three-Level Lambda System

A three-level lambda (Λ) system consists of two lower-energy states, $|1\rangle$ and $|2\rangle$, coupled indirectly through an excited intermediate state $|3\rangle$, which may undergo dissipative decay.

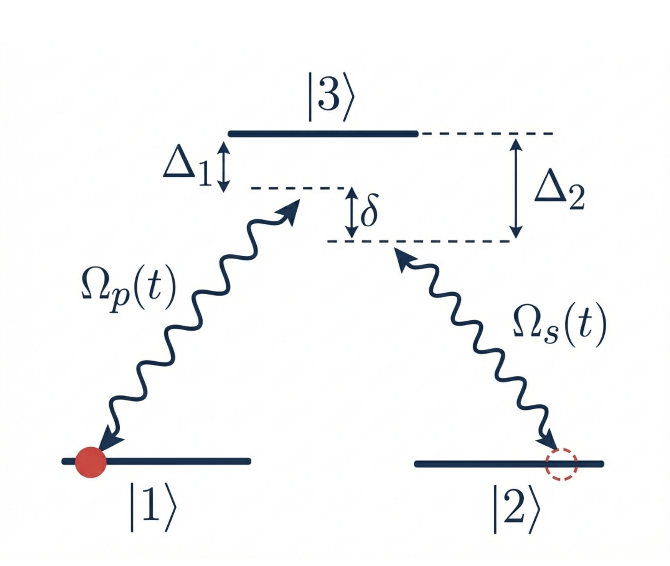

Figure 1. Three level system

Within the rotating-wave approximation, the system is described by the time-dependent Hamiltonian:

$$H_S(t) = \delta\sigma_{22} + \Delta_2\sigma_{33} + \frac{\Omega_p(t)}{2}(\sigma_{31}+\sigma_{13}) + \frac{\Omega_s(t)}{2}(\sigma_{32}+\sigma_{23}) \quad (1)$$

where $\Omega_p(t)$ and $\Omega_s(t)$ are the pump and stokes Rabi frequencies. The parameter Δ corresponds to the one-photon detuning of the excited state $|3\rangle$, while δ denotes the two-photon detuning between $|1\rangle$ and $|2\rangle$. This configuration enables coherent population transfer between $|1\rangle$ and $|2\rangle$ and forms the basis of the STIRAP protocol.
In this work we model $\Omega_p(t)$ and $\Omega_s(t)$ as gaussian pulses:

$$\Omega_s(t) = \Omega_{23}e^{-(t-t_s)^2/2\sigma^2} \quad (2)$$

$$\Omega_p(t) = \Omega_{13}e^{-(t-t_p)^2/2\sigma^2} \quad (3)$$

Dark State: A coherent superposition of the lower states $|1\rangle$ and $|2\rangle$ that does not couple to the excited state $|3\rangle$, suppressing absorption and dissipation. For $\delta = 0$, it is defined as:

$$|\psi_D\rangle = \frac{\Omega_P(t)|2\rangle - \Omega_S(t)|1\rangle}{\sqrt{\Omega_P(t)^2 + \Omega_S(t)^2}} \quad (4)$$

Open Quantum System Dynamics

The system evolution is modeled using the Lindblad master equation to account for **spontaneous decay** from the intermediate state $|3\rangle$ to the ground states $|1\rangle$ and $|2\rangle$. The evolution of the density matrix ρ is governed by:

$$\dot{\rho} = -i[H_S(t), \rho] + \sum_{j=1,2} \mathcal{L}_j(\rho) \quad (5)$$

where $\mathcal{L}_j(\rho) = \gamma_j\left(\sigma_{j3}\rho\sigma_{3j} - \frac{1}{2}\{\sigma_{33}, \rho\}\right)$ are the Lindblad superoperators describing the decay from $|3\rangle \to |j\rangle$ with rate γ_j, and $\sigma_{j3} = |j\rangle\langle 3|$. The initial condition is $\rho(0) = |1\rangle\langle 1|$. This framework allows us to study the robustness of STIRAP under realistic decoherence and decay processes.

Controlability

The system is fully controllable if the **Lie Algebra** generated by the Hamiltonians $\{H_0, H_1, H_2\}$ spans $\mathfrak{su}(3)$. The $\mathfrak{su}(3)$ basis (8 generators) is constructed using the Lie bracket ($[A, B]$).

- **Initial:** $H_0 = \delta\,|2\rangle\langle 2| + \Delta_2\,|3\rangle\langle 3|, \quad H_1 = |3\rangle\langle 1| + |1\rangle\langle 3|, \quad H_2 = |3\rangle\langle 2| + |2\rangle\langle 3|$
- **Raman (Im):** $\mathbf{G_3} = -i[H_1, H_2] = i(|2\rangle\langle 1| - |1\rangle\langle 2|)$.
- **Phases (Im):** $\mathbf{G_4} = -i[H_0, H_1] = -i\Delta_2(|3\rangle\langle 1| - |1\rangle\langle 3|)$. Requirement: $\mathbf{\Delta_2 \neq 0}$.
- **Phases (Im):** $\mathbf{G_5} = -i[H_0, H_2] = -i(\Delta_2 - \delta)(|3\rangle\langle 2| - |2\rangle\langle 3|)$. Requirement: $\mathbf{\Delta_2} \neq \delta$.
- **Raman (Re):** $\mathbf{G_6} = -i[H_0, G_3] = \delta(|2\rangle\langle 1| + |1\rangle\langle 2|)$. Requirement: $\delta \neq \mathbf{0}$.

The set of 8 generators forms a basis for $\mathfrak{su}(3)$ if the non-degeneracy conditions are met. The Λ system is **fully controllable** $\iff \delta \neq \mathbf{0}, \quad \mathbf{\Delta_2 \neq 0}, \quad$ and $\mathbf{\Delta_2} \neq \delta$

Quantum State Engineering Protocol

The Stimulated Raman Adiabatic Passage (STIRAP) protocol achieves robust $|1\rangle \to |2\rangle$ population transfer by leveraging the **Dark State** $|\psi_D\rangle$ properties: it is orthogonal to the excited state $|3\rangle$, thus providing immunity against spontaneous decay. The STIRAP process requires $\Omega_p, \Omega_s \gg \gamma$ (the adiabatic regime). Once this is satisfied, further increasing the amplitudes does not significantly improve the fidelity, the system already follows the dark state adiabatically.

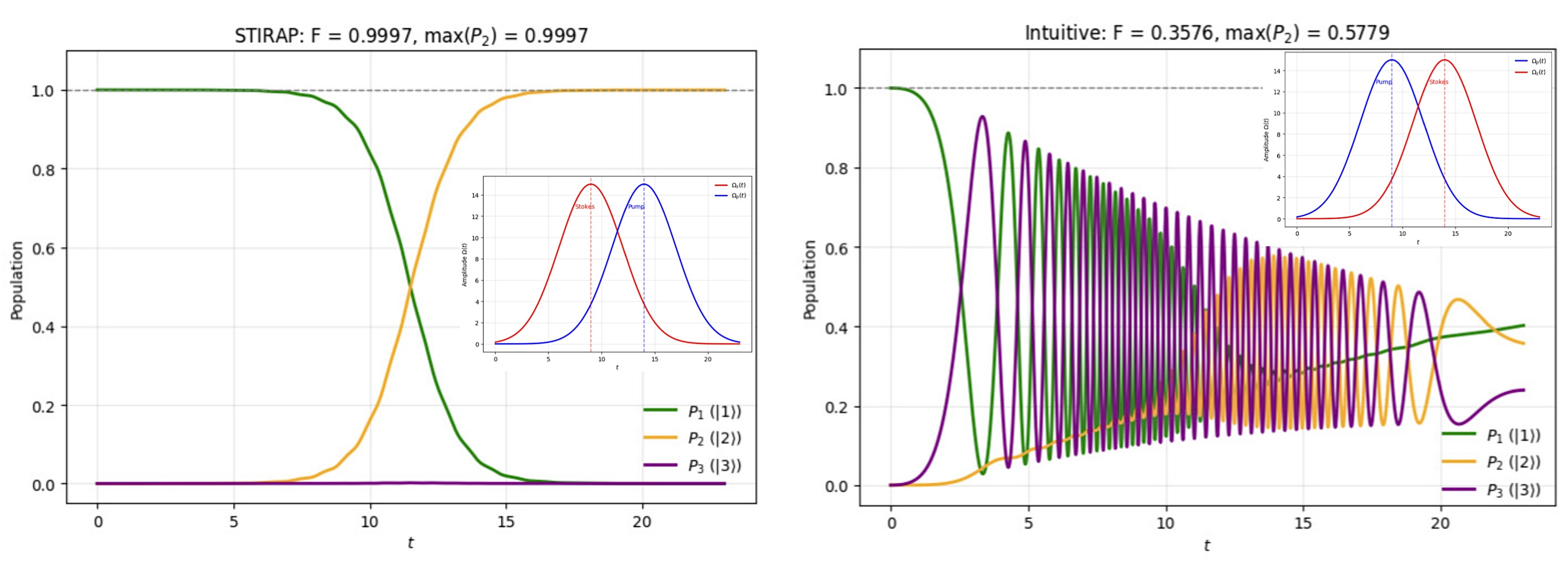

Figure 2. Comparison of counter-intuitive and intuitive approaches

The protocol relies on the **counter-intuitive sequence** where the Stokes pulse (Ω_S), coupling $|2\rangle$ and $|3\rangle$, precedes the Pump pulse (Ω_P), coupling $|1\rangle$ and $|3\rangle$. This sequence drives the adiabatic evolution of the Dark State:

$$|\psi_D(t_{\text{initial}})\rangle \approx |1\rangle \quad \longrightarrow \quad |\psi_D(t_{\text{final}})\rangle \approx |2\rangle \quad (6)$$

Where (ec. 4) is a coherent superposition of the ground states $|1\rangle$ and $|2\rangle$ that **remains decoupled** from the dissipative excited state $|3\rangle$. Population transfer is achieved by implementing the counter-intuitive pulse sequence (Stokes before Pump) with the Rabi frequencies $\Omega_S(t)$ and $\Omega_P(t)$ ($t_s < t_p$). This timing ensures that, throughout the temporal evolution, the Dark State transforms its composition from being predominantly $|1\rangle$ (at the start) to being predominantly $|2\rangle$ (at the end), thus achieving robust coherent transfer and **avoiding population** of the intermediate state $|3\rangle$.

Results and discussion

Fidelity Maps in Parameter Space

Figure 3 shows the fidelity landscape as a function of detuning Δ and decay rate γ, where a wide high-fidelity region is observed. The nominal operating point ($\Delta = 0, \gamma = 0.1$) lies inside this region, indicating robustness against detuning and dissipation. The map in the (A_p, A_s) plane displays an extended high-fidelity plateau, with the optimal point embedded within it. This structure demonstrates that high performance is maintained over a broad range of control amplitudes, without the need for fine parameter tuning.

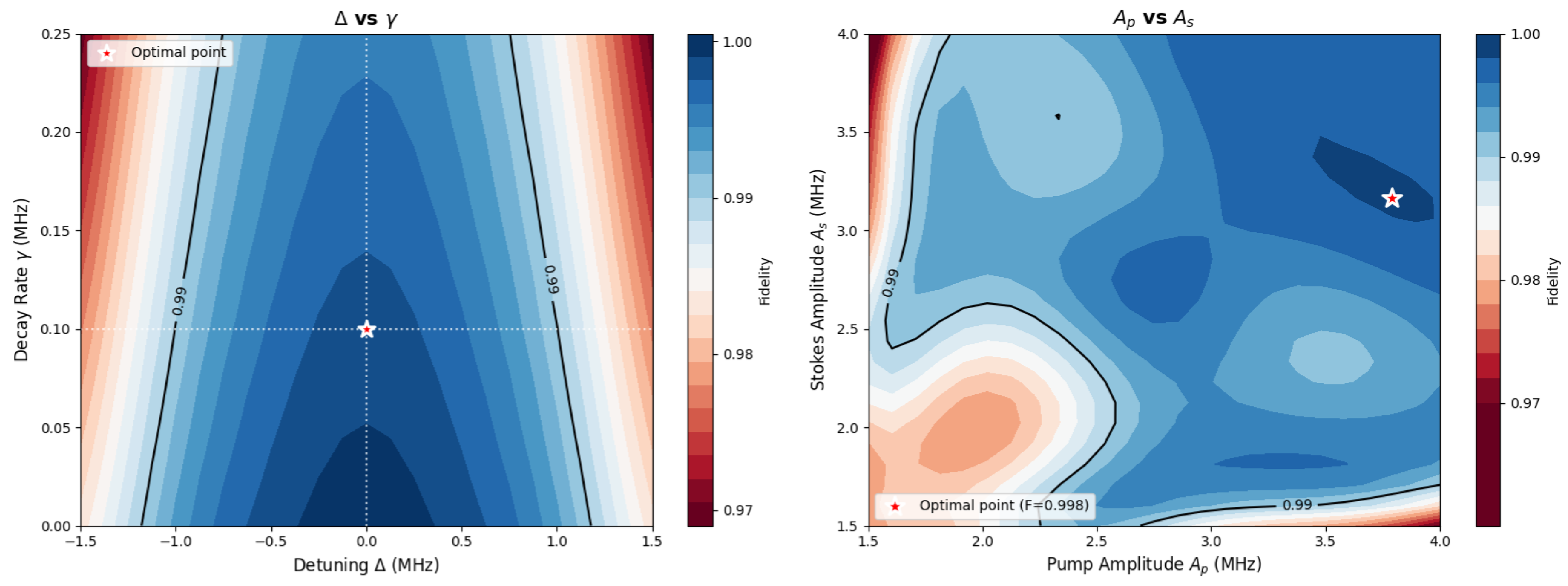

Figure 3. Fidelity maps

Robustness Against Detuning

As shown in Fig.4, the robust optimization yields a higher mean fidelity and a significantly narrower distribution compared to the standard optimization. The histogram shows that the robust solution is strongly concentrated near $F \simeq 0.999$, with a reduced variance ($\sigma \approx 0.001$), while the standard optimization exhibits a broader spread and lower mean fidelity ($\mu \approx 0.996$). This improvement is consistent with the fidelity versus detuning curve, where the robust pulse maintains fidelities above 0.99 over a wider detuning range, whereas the standard solution shows a pronounced degradation for large $|\Delta|$. These results demonstrate that robust optimization effectively suppresses sensitivity to detuning fluctuations.

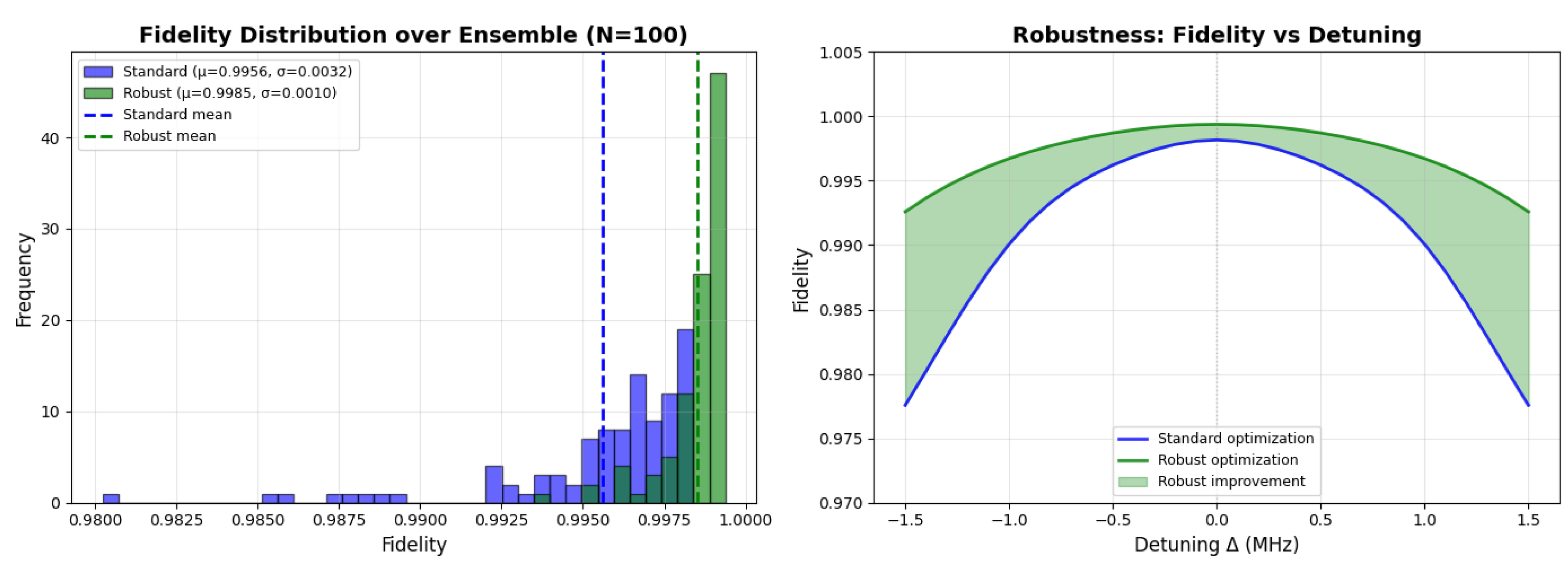

Figure 4. Robustness

Conclusions

- Implementing robust optimization significantly improves the control performance, yielding a higher mean fidelity ($\mu \approx 0.999$) and a markedly narrower fidelity distribution ($\sigma \approx 0.001$) across the ensemble, compared to standard optimization. This demonstrates the effectiveness of robust control techniques in suppressing sensitivity to detuning fluctuations (Δ), thereby ensuring superior and more predictable performance under realistic noisy conditions.
- The adiabatic regime of STIRAP confers intrinsic robustness against control parameter variations. Fidelity maps reveal broad, high-performance plateaus in the pulse amplitude and detuning parameter space, thus minimizing the need for precise parameter tuning in NISQ hardware.

Further Steps

- Investigate the application of CD methods (*Shortcuts to Adiabaticity*) to drastically reduce the STIRAP protocol time. The goal is to accelerate the population transfer while maintaining the dark state's robustness, overcoming the speed limitation imposed by the strictly adiabatic regime.
- Implement and validate the robustly optimized control pulses on a real quantum computer (e.g., **IBM Quantum, IonQ** devices).

References

- K. Bergmann, H. Theuer, and B.W. Shore. Coherent population transfer by stimulated Raman adiabatic passage. *Phys. Rev. A*, 58(4):R1743, 1998.
- H.-P. Breuer and F. Petruccione. *The Theory of Open Quantum Systems*. Oxford Univ. Press, 2007.
- D. D'Alessandro. *Introduction to Quantum Control and Dynamics*. Chapman and Hall/CRC, 2008.
- S. J. Glaser et al. Optimal control of quantum dynamics: theory and experiments. *Phys. Chem. Chem. Phys.*, 17(1):31–48, 2015.
- B. W. Shore. *Manipulation of quantum systems: An introduction to controlling quantum states*. Wiley-VCH, 2008.