XXXI SIMPOSIO PERUANO DE FÍSICA

15 al 19 de diciembre

Wheeler–DeWitt Equation with Non-Minimal Coupling in a Closed FLRW Universe $(k=+1)$

Joseph Cabello Alcarraz* Teofilo Vargas Auccalla
*joseph.cabello@unmsm.edu.pe, *tvargasa@unmsm.edu.pe Universidad Nacional Mayor de San Marcos

INTRODUCTION

Quantum cosmology describes the Universe through the Wheeler-DeWitt equation obtained through canonical quantization of the Hamiltonian constraint. In minisuperspace, a closed FLRW model reduces the problem to a few degrees of freedom, but non-minimal coupling $(\xi R\phi^2)$ alters the kinetic structure and can change the hyperbolic character of the equation.
Using Kiefer's canonical approach, we reduce the Wheeler-DeWitt equation to the closed FLRW case $((k=+1), (\Lambda=0))$ and identify the conditions for hyperbolicity and the emergence of an intrinsic time variable.

MODEL

Consider the Einstein–Hilbert action with non-minimally coupled scalar field plus the Gibbons–Hawking–York edge term [1]

$$S=\int d^4x\,\sqrt{-g}\left[\frac{1}{2}\left(1-\xi\phi^2\right)R-\frac{1}{2}\,g^{\mu\nu}\nabla_\mu\phi\,\nabla_\nu\phi-V(\phi)\right]+S_{\rm GHY}\quad(1)$$

where the metric is considered as follows

$$ds^2=-\,dt^2\;+\;a^2(t)\,d\Omega_3^2,\quad(2)$$

with the spatial term such as

$$d\Omega_3^2=d\chi^2\;+\;\sin^2\chi\left(d\theta^2+\sin^2\theta\,d\varphi^2\right),\quad(3)$$

$$0\le\chi\le\pi.\quad(4)$$

After applying the ADM decomposition in the Hamiltonian formulation, a separation of kinetic terms must be carried out, for this purpose the redefinition of the field is used using the following ansatz

$$\chi=\sqrt{2}\pi\phi a^{6\xi}\quad(5)$$

REDUCED LAGRANGIAN

With the previous result, after applying the ansatz, the following Lagrangian is obtained that depends on the scale factor and the rescaling term of the scalar field $\mathcal{L}(a,\chi)$

$$L(a,\chi)=-6\pi^2\,a\,\dot a^{\,2}\;+\;6\pi^2\,a+\frac{1}{2}\,a^{3-12\xi}\,\dot\chi^{\,2}+3\xi(1-6\xi)\,a^{\,1-12\xi}\,\dot a^{\,2}\,\chi^2-3\xi\,a^{\,1-12\xi}\,\chi^2-2\pi^2\,a^3\,V\!\left(\frac{\chi}{\sqrt{2}\,\pi\,a^{6\xi}}\right).\quad(6)$$

There are no mixed terms after the redefinition of the field. This is important for constructing the Lagrangian and subsequently finding solutions.
The effective coupling depends on χ and ξ

HAMILTONIAN

What is sought is for the Wheeler-DeWitt equation to have hyperbolic behavior; for this the Hamiltonian must meet the following condition

$$\mathcal{H}=0\qquad,\qquad\hat p\to-i\hbar\partial\quad(7)$$

$$[A(a,\chi)\partial_a^2-B(a,\chi)\partial_\chi^2+U(a,\chi)]\Psi(a,\chi)=0\quad(8)$$

The resulting action can be written as

$$S=\frac{1}{2}\int dt\left[-6a\,\dot a^{\,2}\left(\frac{1}{48\pi G}+\xi\Big(\xi-\frac{1}{6}\Big)\chi^2a^{-12\xi}\right)+a^{3-12\xi}\,\dot\chi^{\,2}+\frac{a}{8\pi G}-6\xi\,\chi^2a^{1-12\xi}-m^2\chi^2a^{3-12\xi}\right].\quad(9)$$

and then calculating the conjugate moments in function of $\pi_a=\partial\mathcal{L}/\partial\dot a$ and $\pi_\chi=\partial\mathcal{L}/\partial\dot\chi$

WHEELER-DEWITT EQUATION

Substituting the above results into the Legendre transform to obtain the Hamiltonian with the restriction in terms of $(a,\chi;\pi_a,\pi_\chi)$

$$H=\frac{1}{2}\left[\frac{\pi_a^2}{6\xi(1-6\xi)\chi^2a^{1-12\xi}-\dfrac{a}{8\pi G}}\;+\;a^{12\xi-3}\,\pi_\chi^2\right]-\frac{1}{2}\left[\frac{a}{8\pi G}-6\xi\,\chi^2a^{1-12\xi}-m^2\chi^2a^{3-12\xi}\right]=0\quad(10)$$

CONCLUSIONS

- The restricted Hamiltonian $H=0$ leads to a Wheeler–DeWitt equation whose kinetic structure is governed by the "metric" of the minisuperspace $(a,)$. In particular, the sign of the gravitational component controls the type of equation (hyperbolic vs. elliptic).
- If $\xi=1/12$, the type change is associated with a boundary defined by an algebraic condition at (a,), which separates classically allowed and forbidden regions, and allows discussion of phenomena such as "phase space tunnelling".

REFERENCES

[1] Valerio Faraoni. «Nonminimal Coupling». en. En: Fundamental Theories of Physics (2004). Ed. por Valerio Faraoni, págs. 143-195. DOI: 10.1007/978-1-4020-1989-0_7. (Visitado 01-12-2023).