# Generalization of Quantum Decay Laws and Spectral Decomposition via a Canonical Time Operator in Deformed Algebras Induced by Gravitational Effects

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**Abstract.** We study how the generalization of the energy–time uncertainty principle (GUP), produced by quantum gravity effects, modifies the decay laws of quantum systems. Since the energy–time uncertainty relation constrains the behavior of these laws, modifying this relation naturally implies a modification of the decay laws, so a generalized version of the Fock–Krylov formalism is constructed, which allows the classical decay laws to be modified. The modifications experienced by the Lorentzian spectrum are analyzed, obtaining its modified decay law as well as a relation between the GUP mean lifetime and the classical lifetime, where the corrections are observed to correspond to Planck times. Finally, under the assumption that the Stefan–Boltzmann law remains valid and using the effective temperatures of a micro black hole, the modification to the evaporation time of a Schwarzschild black hole is estimated.

## 1 Introduction

At scales close to the Planck scale, quantum gravity theories suggest that effects arise which ordinary quantum mechanics does not account for; the generalized uncertainty principle (GUP) allows these effects to be incorporated through the modified version of the energy–time uncertainty relation, which helps us better understand the effects occurring at these scales. In particular, if the decay of an unstable state depends on its energy dispersion, such a correction should alter the way it decays. This work aims to approximate these effects and to contribute to understanding how quantum systems decay at Planck scales.

## 2 Methodology

We start from the generalized uncertainty principle

$$\Delta t\,\Delta E \;\geq\; \frac{\hbar}{2}\big[1+\alpha(\Delta E)^2\big],$$

obtained from the deformed algebra $[\hat{T},\hat{E}] = i\hbar(1+\alpha\hat{E}^2)$. From this commutation relation, an energy representation with modified inner product and integration measure is constructed, following the structure developed in [2]. In the temporal sector, the deformation implies the existence of a minimum measurement interval $\Delta t_{\min} = \hbar\sqrt{\alpha}$, below which it is not possible to obtain information about the system, which leads to a modified time-evolution equation [3]. With these tools, a generalized version of the Fock–Krylov formalism is constructed [1], in which the energy spectrum $P(E)$ is weighted by the factor $(1+\alpha E^2)^{-1}$, so the survival amplitude takes the form of a corrected integral over this spectrum. To analyze the consequences on decay, a Breit–Wigner spectrum is considered.

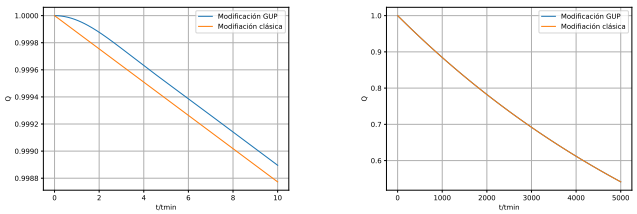
For the black hole case, a thermodynamic approach is adopted, in which, starting from the modified temperatures of a micro black hole obtained in [4], and assuming that the Stefan–Boltzmann law remains valid, its lifetime is estimated.

## 3 Results and discussion

From the modified Fock–Krylov formalism applied to a Lorentzian spectrum, the decay law

$$Q(x) = \frac{N^2}{\pi^2}\sin^2\!\left(\frac{\pi x}{2}\right)\left\{-\frac{4}{x(x^2-4)}\right\}^2,$$

is obtained, with $x = t/t_{\min}$. For the simple case in which $\Gamma = 2/\sqrt{\alpha}$, comparing this expression with the classical exponential law shows appreciable deviations at short times, while at long times both converge (Fig. 1).




**(a)** Short times. **(b)** Long times.

**Figure 1:** Behavior of $Q(t)$ for the Breit–Wigner distribution with $\Gamma = 2.495 \times 10^{24}$ eV: (a) short times, (b) long times.

The mean lifetime of this spectrum exhibits the correction

$$\tau_{\mathrm{GUP}} \;\simeq\; \tau_{\mathrm{cl}} + 0.6004\,t_P,$$

where $\tau_{\mathrm{cl}} = \hbar/\Gamma$ and $t_P \simeq 5.39\times10^{-44}$ s is the Planck time. This correction is negligible at macroscopic scales and becomes relevant only for very short decay times or large energy dispersions.

For black hole evaporation, the thermodynamic model leads to the estimate

$$t_{\pm} \;\simeq\; \tau_H\left[1-\frac{3\alpha_{\pm}}{64}\left(\frac{\hbar c^3}{GM_0}\right)^2\right].$$

For large masses the correction tends to zero and the classical behavior is recovered; only for masses close to the Planck mass does the deformation produce appreciable effects.

## References

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