Statistical effect in the parton distribution functions of the nucleon

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A B S T R A C T

A new and simple statistical approach is performed to calculate the parton distribution functions (PDFs) of the nucleon in terms of light-front kinematic variables. We do not put in any extra arbitrary parameter or corrected term by hand, which guarantees the stringency of our approach. Analytic expressions of the x-dependent PDFs are obtained in the whole x region [0, 1], and some features, especially the low-x rise, are more agreeable with experimental data than those in some previous instant-form statistical models in the infinite-momentum frame (IMF). Discussions on heavy-flavored PDFs are also presented.

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1. Introduction

One of the goals in fundamental physics is to search for the detail information of the nucleon structure. The parton model, suggested by Feynman [1], and then immediately applied to the deep inelastic lepton–nucleon scattering process by Bjorken [2], proposes that the nucleon is composed of a number of point-like constituents, named "partons", which were afterward recognized as quarks and gluons. In the impulse approximation, the deep inelastic lepton–nucleon scattering can be viewed as a sum of elastic lepton–parton scattering, in which the incident lepton is scattered off a parton instantaneously and incoherently. This is in accord with one property of QCD – asymptotic freedom. On the other hand, due to another property of QCD – color-confinement, the constituents of nucleon – quarks and gluons, have never been seen individually. The nucleon structure functions, in terms of the parton distribution functions (PDFs), are badly desired in hadronic study. However, due to the complicated non-perturbative effect, we still have difficulty to calculate them absolutely from the first principal theory of QCD at present.

Various models according to the spirit of QCD have been brought forward, therein statistical ones, providing intuitive appeal and physical simplicity, have made amazing success [3–21]. Actually, as can be speculated, with partons bound in the wee

volume of the nucleon, not only the dynamic, but also the statistical properties, for example, the Pauli exclusion principle, should have important effect on the PDFs. Cleymans and Thews [3,4], as pioneers, started with the transition rate of scattering in the framework of temperature dependent field theory and explored a statistical way to generate compatible PDFs. Mac and Ugaz [5] incorporated first order QCD corrections (however, the perturbative term turned out to be a sizable fraction of the statistical term), and afterwards Bhalerao et al. [11–14] introduced finite-size correction and got more fitting results; they both referred to the infinite-momentum frame (IMF). Bickerstaff and Londergan [6] interpreted the finite-temperature property as to mimic some of the volume-dependent effects due to confinement, furthermore, they discussed the theoretical validity of the ideal gas assumption in detail. Devanathan et al. [8–10] proposed a thermodynamical bag model which evolves as a function of x, and the structure functions they got are practicable for $x > 0.2$ and have correct asymptotic behavior as $x \to 1$; in addition, they parametrized on T and exhibited the scaling behavior. Bourrely et al. [15–17] developed a new form of statistical parametrization, allowing x-dependent chemical potential, and by incorporating QCD evolution they got indeed remarkable PDFs. Otherwise, Zhang et al. [18–20] constructed a model using the principle of detailed balance and balance without any free parameter, and the Gottfried sum they got is in surprisingly agreement with experiments. Alberg and Henley [21] tracked the detailed balance model for a hadron composed of quark and gluon Fock states and obtained parton distributions for the proton as well as the pion.

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When dealing with physics in high energy region, light-front dynamics is a suitable language [22]. It is known [23] that the impulse approximation fails when using instant-form dynamics in the nucleon rest frame, but works well when using it in the IMF or using light-front dynamics in an ordinary frame. However, statistics in light-front formalism encounters some difficulties, therein the most fatal one is the unclarity about what the light-front temperature is and how it relates with the usual one in instant-form formalism (see, e.g., Refs. [24–27]). Consequently, the way of generalization from instant-form statistics to light-front statistics is quite speculative. Actually, even the generalization of the thermal dynamics and statistical theory from the system rest frame to a moving frame in instant-form formalism is also not so understood, and discussion on it has continued for a long period (for theoretical discussion, see Refs. [28–34] and references therein, and for recent numerical experiments, see, e.g., Refs. [35,36]).

In order to avoid these tough problems, we start with instant-form statistical expressions in the nucleon rest frame, then perform transformation on them in terms of light-front kinematic variables. The analytic expressions of the PDFs we get are something different from those attained in other statistical models performing in the IMF [5,11–14]. The largest distinction is that, when $x \to 0$, the distributions of our light-flavored quarks (anti-quarks) do not tend to zero as theirs, but give a rise instead, which agrees with the experimental data better.

Worthy to note that, our intention is only to illustrate whether the statistical effect is important and to which aspects of the nucleon structure it is important, not how well it matches the experimental results, so we do not make any effort to fit the experimental data intentionally. There is no arbitrary parameter put by hand in our model, and all parameters are basic statistical quantities. Some of other statistical models can fit the experimental data better by introducing many free parameters, however, it weakens the stringency at a cost. In addition, our results naturally cover the whole x region [0, 1], and the features of PDFs and structure functions at the boundary are of great interest in both theoretical and experimental study.

The Letter is organized as follows. In Section 2, a brief description about the approach used in this Letter is introduced, and the analytic expressions of the PDFs are presented. In Section 3, numerical results and comparisons with experiments and other theories are illustrated. In Section 4, the mass effect of the partons and the features of heavy-flavored PDFs are discussed. Section 5 is a short summary.

2. The statistical approach

We assume that the nucleon is a thermal system in equilibrium, made up of free partons (quarks, anti-quarks and gluons). In the nucleon rest frame, the mean number of the parton (denoted by f) is

$$\bar{N}_f = \int f(k^0) \, \mathrm{d}^3 k, \tag{1}$$

where $f(k^0)$ satisfies the Fermi–Dirac distribution or the Bose–Einstein distribution

$$f(k^0) = \frac{g_f V}{(2\pi)^3} \frac{1}{e^{\frac{k^0 - \mu_f}{T}} \pm 1}, \tag{2}$$

with the upper sign for Fermion (quark, anti-quark), and nether sign for Boson (gluon); g_f is the degree of color-spin degeneracy, which is 6 for quark (anti-quark) and 16 for gluon; μ_f is its chemical potential, and for anti-quark $\mu_{\bar{q}} = -\mu_q$, for gluon $\mu_g = 0$.

Instead of boosting above expressions to the IMF [5,11–14], we transform them in terms of light-front kinematic variables in the nucleon rest frame. Before doing this, note that when

doing the integration in Eq. (1), the on-shell condition $k^0 = \sqrt{\mathbf{k}^2 + m_f^2}$ is needed, where k^0, $\mathbf{k} = (k^1, k^2, k^3)$, m_f are the energy, 3-momentum and mass of the parton, respectively. Eq. (1) can be explicitly reexpressed as

$$\bar{N}_f = \int f(k^0) \delta\big(k^0 - \sqrt{(k^3)^2 + \mathbf{k}_\perp^2 + m_f^2}\big) \mathrm{d}k^0 \, \mathrm{d}k^3 \, \mathrm{d}^2 k_\perp. \tag{3}$$

We introduce the light-front 4-momentum of the parton $k = (k^+, k^-, \mathbf{k}_\perp)$, where $k^+ = k^0 + k^3$, $k^- = k^0 - k^3$, $\mathbf{k}_\perp = (k^1, k^2)$, and $k^+ = P^+ x = Mx$, where x is the light-front momentum fraction of the nucleon carried by the parton and M is the mass of the nucleon. Hereby, the δ-function and the integral in Eq. (3) turn to

$$\delta\big(k^0 - \sqrt{(k^3)^2 + \mathbf{k}_\perp^2 + m_f^2}\big)$$
$$= 2k^0 \theta(k^0) \delta(k^2 - m_f^2)$$
$$= \left[1 + \frac{\mathbf{k}_\perp^2 + m_f^2}{(Mx)^2}\right] \theta(x) \delta\left(k^- - \frac{\mathbf{k}_\perp^2 + m_f^2}{Mx}\right) \tag{4}$$

and

$$\mathrm{d}k^0 \, \mathrm{d}k^3 \, \mathrm{d}^2 k_\perp = \frac{1}{2} M \, \mathrm{d}k^- \, \mathrm{d}x \, \mathrm{d}^2 k_\perp. \tag{5}$$

Then Eq. (3) becomes

$$\bar{N}_f = \int f(x, \mathbf{k}_\perp) \, \mathrm{d}^2 k_\perp \, \mathrm{d}x, \tag{6}$$

where $f(x, \mathbf{k}_\perp)$ is

$$f(x, \mathbf{k}_\perp) = \frac{g_f M V}{2(2\pi)^3} \frac{1}{e^{\frac{\frac{1}{2}(Mx + \frac{\mathbf{k}_\perp^2 + m_f^2}{Mx}) - \mu_f}{T}} \pm 1} \left[1 + \frac{\mathbf{k}_\perp^2 + m_f^2}{(Mx)^2}\right] \theta(x). \tag{7}$$

On the trivial assumption that $\mathbf{k}_\perp$ is transversely isotropic, we can integrate on $|\mathbf{k}_\perp|$ analytically and get

$$f(x) = \pm \frac{g_f M T V}{8\pi^2} \left\{ \left(Mx + \frac{m_f^2}{Mx}\right) \ln\left[1 \pm e^{-\frac{\frac{1}{2}(Mx + \frac{m_f^2}{Mx}) - \mu_f}{T}}\right] \right.$$
$$\left. - 2T \, \mathrm{Li}_2\big(\mp e^{-\frac{\frac{1}{2}(Mx + \frac{m_f^2}{Mx}) - \mu_f}{T}}\big) \right\}, \tag{8}$$

as is mentioned above, the upper sign for Fermion and the nether sign for Boson. $\mathrm{Li}_2(z)$ is the polylogarithm function, defined as $\mathrm{Li}_2(z) = \sum_{k=1}^{\infty} z^k / k^2$. Note that the expressions of the PDFs (Eq. (8)) are different from those attained in the previous statistical models [3–5,7,11–14].

In practice, the PDFs in a certain system should be constrained with some conditions. For example, in the proton, they are

$$u_V = \int_0^1 \big[u(x) - \bar{u}(x)\big] \, \mathrm{d}x = 2, \tag{9}$$

$$d_V = \int_0^1 \big[d(x) - \bar{d}(x)\big] \, \mathrm{d}x = 1, \tag{10}$$

$$\int_0^1 \big[u(x) + \bar{u}(x) + d(x) + \bar{d}(x) + g(x)\big] x \, \mathrm{d}x = 1. \tag{11}$$

Thus for the proton, we have three Eqs. (9), (10), (11), and four unknown parameters T, V, μ_u, μ_d (the mass of the proton, $M = 938.27$ MeV, is taken as given). So for a given T, the rest parameters V, μ_u, μ_d can be determined by solving the equations.

3. Results

We perform our calculation for the proton, therefore the following results, if not specially stated, are all for proton. However, the method is absolutely applicable to the neutron. For convenience, we just consider the u, d flavor and gluon, and take $m_f = 0$, which will be showed, in Section 4, as a good approximation.

In practice, we adopt a certain value of T at first, then numerically solve the equations to get V, μ_u, μ_d. Subsequently, the PDFs can be obtained according to Eq. (8), as well as the Gottfried sum

$$S_G = \int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x}\, dx = \frac{1}{3} + \frac{2}{3} \int_0^1 \left[\bar{u}(x) - \bar{d}(x) \right] dx. \tag{12}$$

We find that at $T_0 = 47$ MeV, the Gottfried sum $S_G = 0.236$, which agrees well with the experimental result $S_G = 0.235 \pm 0.026$ [37]. We conclude that the temperature of proton is around 47 MeV, and $V_0 \approx 1.2 \times 10^{-5}$ MeV^{-3}, $\mu_u \approx 64$ MeV, $\mu_d \approx 36$ MeV. The following results are all given at $T_0 = 47$ MeV.

Taking proton as a perfect sphere, we can calculate its radius r_0 from the volume V_0. We get $r_0 = 2.8$ fm, which seems a little bigger than the practical value, possibly due to the oversimplified assumption of the uniform distribution of partons. Worthy to mention that, the r_0 we get, together with T_0, is close to what Mac and Ugaz [5] got with the consideration of first-order QCD corrections, and their fitted values are $T = 49$ MeV, $r = 2.6$ fm.

The PDFs $f(x)$ and $xf(x)$ are illustrated in Figs. 1 and 2, respectively. In contrast to $q(x)$ and $\bar{q}(x) \to 0$ as $x \to 0$ in the previous statistical models without extra corrected term [3–5,7], our trend makes a good improvement.

In our model, the flavor asymmetry of the nucleon sea is naturally generated. $\bar{d}(x) - \bar{u}(x)$ and $\bar{d}(x)/\bar{u}(x)$ are shown in Figs. 3 and 4, the former agrees well with the results of experiments and CTEQ parametrization [38] while the latter seems not. The model suggested by Zhang et al. [18–20] also gives reasonable asymmetry of $\bar{u}$ and $\bar{d}$ without introducing any parameter, which is further discussed by Alberg and Henley [21]. And the feature of $\bar{d}(x) - \bar{u}(x)$ in Refs. [19,21] and $\bar{d}(x)/\bar{u}(x)$ in Ref. [21] have similar behavior as ours.

Furthermore, $d(x)/u(x)$ is also shown in Fig. 5. Note that when $x \to 1$, $d(x)/u(x) \to 0.55$, which is different from the result of CTEQ parametrization [38], but close to the prediction of the naive SU(6) quark model.

The nucleon structure function $F_2(x) = 2xF_1(x) = x \sum_f e_f^2 f(x)$, where e_f is the charge of the parton of flavor f, is shown in Fig. 6. With the p–n isospin symmetry, i.e. $u^n(x, \mathbf{k}_\perp) = d^p(x, \mathbf{k}_\perp)$, $d^n(x, \mathbf{k}_\perp) = u^p(x, \mathbf{k}_\perp)$, $\bar{u}^n(x, \mathbf{k}_\perp) = \bar{d}^p(x, \mathbf{k}_\perp)$, $\bar{d}^n(x, \mathbf{k}_\perp) = \bar{u}^p(x, \mathbf{k}_\perp)$,
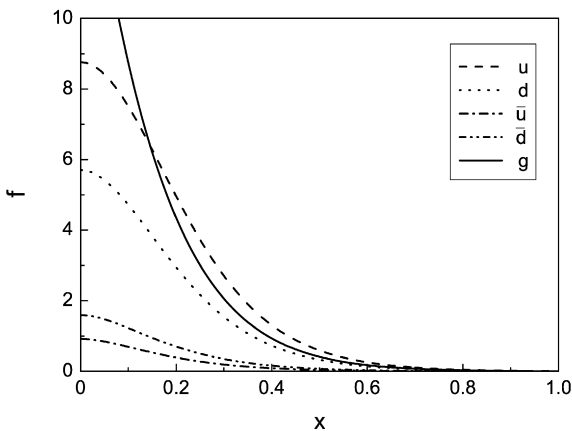
$g^n(x, \mathbf{k}_\perp) = g^p(x, \mathbf{k}_\perp)$, we can also obtain the structure function of the neutron $F_2^n(x)$. $F_2^p(x) - F_2^n(x)$ and $F_2^n(x)/F_2^p(x)$ are shown in Figs. 7 and 8. We can see that $F_2^p(x) - F_2^n(x)$ is quite agreeable, while $F_2^n(x)/F_2^p(x)$ is not. One interesting feature is the behavior of $F_2^n(x)/F_2^p(x)$ when $x \to 1$. In the naive SU(6) quark model it tends to 2/3, while in the SU(6) quark–diquark model it is 1/4 and in experimental observation it is smaller than 1/2. Here our result seems close to the prediction of the naive SU(6) quark model again, however, it does not agree with the reality.
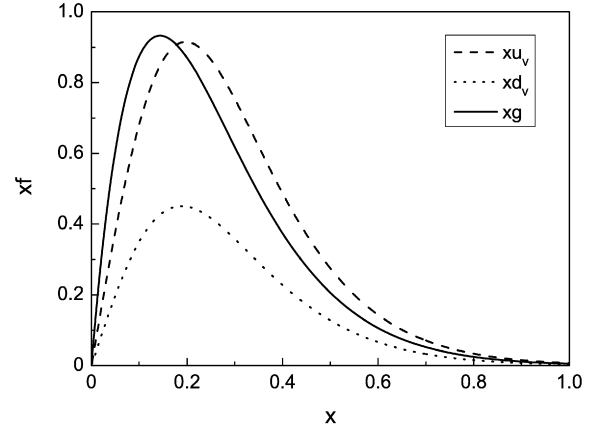

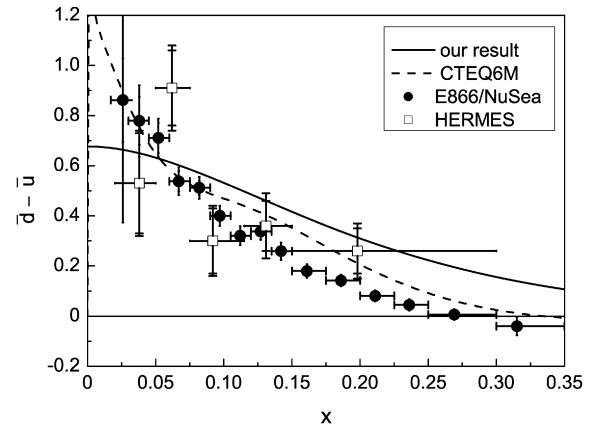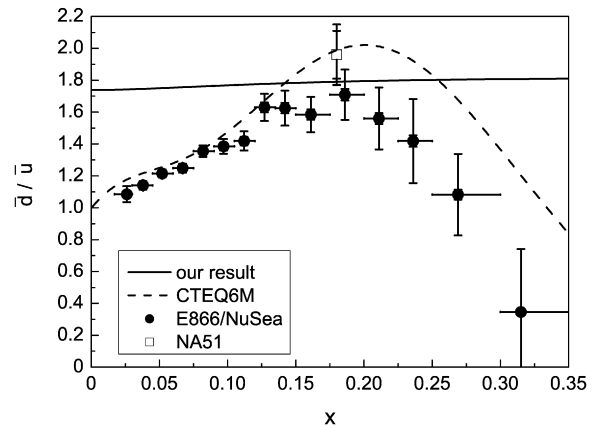**Fig. 2.** The calculated $xf(x)$ in our statistical approach.

Fig. 3. Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38], E866/NuSea result at $Q^2 = 54$ GeV^2 [39] and HERMES result at $\langle Q^2 \rangle = 2.3$ GeV^2 [41] for $\bar{d}(x) - \bar{u}(x)$.

Fig. 4. Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38], E866/NuSea result at $Q^2 = 54$ GeV^2 [39] and NA51 result [40] for $\bar{d}(x)/\bar{u}(x)$.

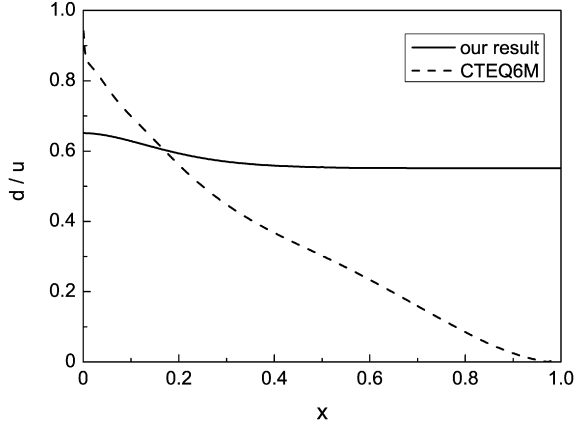**Fig. 1.** The calculated $f(x)$ in our statistical approach.

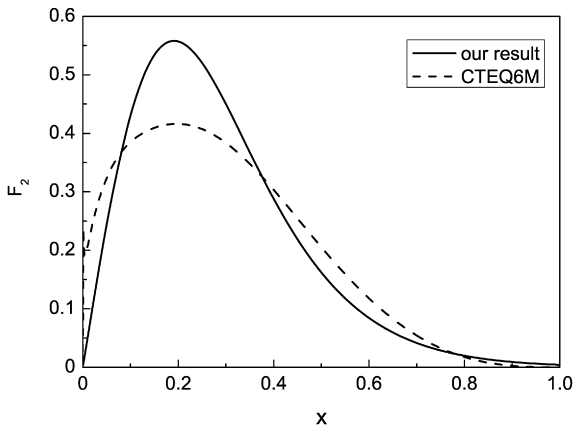**Fig. 5.** Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38] for $d(x)/u(x)$.

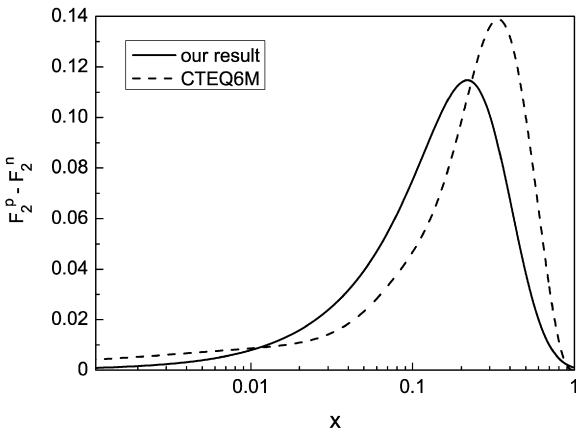**Fig. 6.** Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38] for $F_2(x)$.

Fig. 7. Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38] for $F_2^p(x) - F_2^n(x)$.

From the above results, we find that our statistical method can successfully describe the behavior of the "subtracted-terms", such as $\bar{d}(x) - \bar{u}(x)$ and $F_2^p(x) - F_2^n(x)$, but the "divided-terms", such as $\bar{d}(x)/\bar{u}(x)$, $d(x)/u(x)$ and $F_2^p(x)/F_2^n(x)$, can only match the trend of experimental results approximately, and the departure is especially large in the high-x region, where the valence parts of the PDFs dominate. This feature probably implies that an additive statistics-irrelevant corrected term to the PDFs works, whereas more free parameters and uncertainty should be introduced. Bhalerao et al. successfully reproduced most features of the PDFs and structure functions by introducing two extra corrected terms [11–14], at the
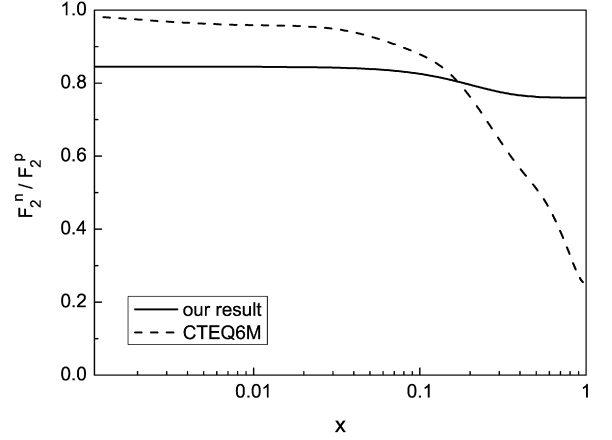

Fig. 8. Comparison of our result with CTEQ result at $Q^2 = 1$ GeV^2 [38] for $F_2^p(x)/F_2^n(x)$.

cost of two more free parameters and violating the p–n isospin symmetry.

4. Further discussions

We have ignored the masses of the quarks and anti-quarks for simplicity. Nevertheless, mass effect can be taken into account without difficulty. Actually we have performed this calculation and found, as can be speculated, the correction of mass effect to the light-flavored PDFs is very small. $\bar{d}(x) - \bar{u}(x)$ and $d(x)/u(x)$ with different masses of u, d quark are illustrated in Fig. 9.

However, the mass difference between u and d quarks can generate the mass split between the proton and the neutron. The invariant mass square of the system is given by

$$M^2 = \sum_i \left(\frac{m^2 + k_\perp^2}{x} \right)_i. \tag{13}$$

In the continuous condition, it is

$$M^2 = \sum_f \int \left[\iint \left(\frac{m_f^2 + k_\perp^2}{x} \right) f(x, k_\perp) \, d^2 k_\perp \right] dx. \tag{14}$$

Using the p–n isospin symmetry, we get

$$M_n^2 - M_p^2 = (m_d^2 - m_u^2) \int \frac{1}{x} \big[u^p(x) + \bar{u}^p(x) - d^p(x) - \bar{d}^p(x) \big] dx. \tag{15}$$

In PDG 2006, $m_u = 1.5 \sim 4$ MeV, $m_d = 3 \sim 7$ MeV, $\Delta_{pn} = M_n - M_p = 939.565 - 938.272 = 1.293$ MeV.

When we use the mean value $m_u = 2.25$ MeV, $m_d = 5$ MeV, we get $\Delta_{pn} = 0.664$ MeV from Eq. (15), and when we use the extreme value $m_u = 1.5$ MeV, $m_d = 7$ MeV, then $\Delta_{pn} = 1.557$ MeV. The result seems rather agreeable and it reinforces the reasonableness of our approach.

We have only calculated the light-flavored PDFs, however, the heavy flavors, such as s, c, b, t quarks and the corresponding anti-quarks, can be treated in the same way. Since, in nucleon, the valence numbers of them are zero, the chemical potentials of them must all be zero. It leads directly to three following conclusions:

Firstly, except for their masses, the heavy-flavored PDFs have no additional free parameters than the light-flavored ones. That is, if their masses are used as inputs, their PDFs can be uniquely determined by the parameters T and V, which have already been fixed in the previous light-flavored condition. So the difference between heavy-flavored PDFs only comes from the difference of their masses.
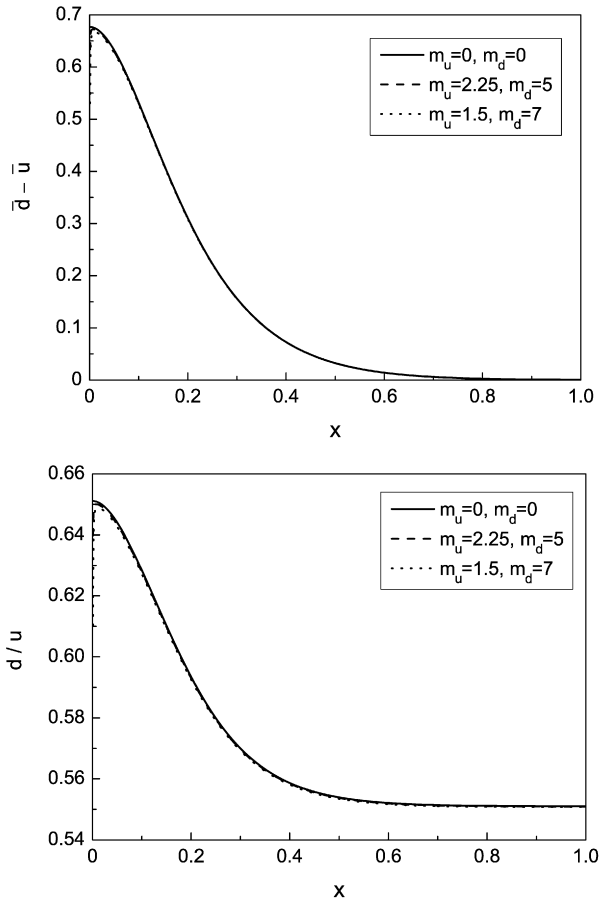
Fig. 9. $\bar{d}(x) - \bar{u}(x)$ and $d(x)/u(x)$ at different masses (unit: MeV) of u, d quark.
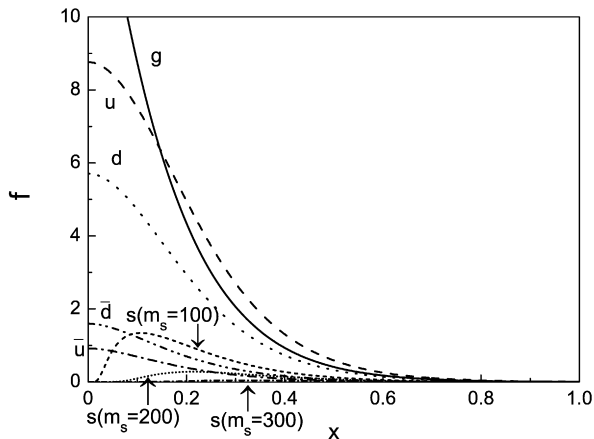

Fig. 10. Comparison of $s(x)$ at different mass ($m_s = 100$, 200, 300 MeV) with the light-flavor PDFs at T_0 and V_0.

Secondly, the quark and anti-quark of the same heavy flavor have just the same distribution. For example, the condition $s(x) = \bar{s}(x)$ holds in the whole region $x \in [0, 1]$, so that the s, $\bar{s}$ asymmetry in the nucleon [42] does not come from the pure statistical effect.

Thirdly, from Eq. (8), we can see that $f(x)$ decreases when m_f increases. $s(x)$ with different m_s at T_0 and V_0, together with the light-flavored PDFs, are illustrated in Fig. 10, and it shows that when $m_s \leqslant 100$ MeV the contribution of s quark is considerable, and when $m_s > 200$ MeV it is minor. Therefore, the contribution of heavier flavors is negligible. Calculation also indicates that s quark contributes less than 7% both to the total light-front momentum

fraction x (see Eq. (11)) and to the total invariant mass square of the system at T_0 and V_0 (see Eq. (14)).

5. Summary

We preform a new statistical approach and obtain analytic expressions of the parton distribution functions in terms of light-front kinematic variables in the whole x region [0, 1]. The low-x behavior of these parton distribution functions is different from those in some previous instant-form statistical models in the infinite-momentum frame and our results are more close to the reality. There is no arbitrary parameter or extra corrected term put by hand in our model, which guarantees the stringency of our conclusion. Several features of the parton distribution functions and structure functions of the nucleon are compared with the results of experiments and other theories. Calculation of the mass split between the proton and the neutron is also performed. We have further discussions on the influence of the heavy flavors. All of these show that although the statistical effect is not everything, it is very important to some aspects of the nucleon structure.

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