Analogue of superradiance effect in acoustic black hole in the presence of disclination

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Curved space Klein Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)\phi = 0, \tag{1}$$

where $g_{\mu\nu}$ is a metric tensor (with Lorentizian signature), not of spacetime itself. but an acoustic "analog spacetime". In this model, the velocity potential, in polar coordinates is given by

$$\phi(r,\theta) = A\log r + B\theta, \tag{2}$$

where A and B are real constants and ϕ present a sink in the origin. This leads to the velocity profile

$$\vec{v} = \frac{A}{r}\hat{r} + \frac{B}{r}\hat{\theta}, \tag{3}$$

The geometric with a disclination has the line element given by

$$\begin{aligned} ds^2 = -\left(1-\frac{A^2+B^2}{c^2r^2}\right)dt^2 - \left(1-\frac{A^2}{c^2r^2}\right)^{-1}dr^2 \\ -2\frac{B}{c}\alpha d\theta dt + r^2\alpha^2 d\theta^2 + dz^2, \end{aligned} \tag{4}$$

Remove $(0 < \alpha \leq 1)$ or insert $(2\pi > \alpha \geq 1)$ a wedge of material of dihedral angle $\lambda = 2\pi(\alpha - 1)$. Now, we write the Klein-Gordon equation in the background metric and we can separate variables by the substitution

$$\phi(t, r, \theta) = \exp i(\omega t - m\theta) R(r),$$

where m is an integer, we assume that $\omega > 0$, then, the radial function satisfies the equation given by

$$\frac{d^2 g(r^*)}{dr^{*2}} + \omega^2 g(r^*) = 0, \tag{5}$$

where $g(r^*) \equiv r^{1/2} R(r)$, and considering $r \to \infty$, introducing the tortoise coordinate r^* we obtain

$$g(r^*) = \exp(i\omega r^*) + \mathcal{R} \exp(-i\omega r^*). \tag{6}$$

The first term of eq. (6) corresponds to an ingoing wave and the second term corresponds to de reflected wave, where $\mathcal{R}$ is the reflection coefficient.

Wronskian of the solutions (6) given by

$$\mathcal{W}(+\infty) = -2i\omega\left(1 - |\mathcal{R}|^2\right). \tag{7}$$

Thus, near the horizon $r^* \to -\infty$, we obtain

$$\frac{d^2 g(r^*)}{dr^{*2}} + \left(\omega - m\Omega_{H,\alpha}\right)^2 g(r^*) = 0 \tag{8}$$

where $\Omega_{H,\alpha} \equiv \frac{Bc}{\alpha A^2}$ is the angular velocity of the acoustic black hole in the presence of a disclination. Near the horizon, we suppose that just the solution identified by ingoing wave is physical, is that

$$g(r^*) = \mathcal{T}\exp[i\left(\omega - m\Omega_{H,\alpha}\right) r^*], \tag{9}$$

where $\mathcal{T}$ is the transmission coefficient. So,

$$\mathcal{W}(-\infty) = -2i\left(\omega - m\Omega_{H,\alpha}\right)|\mathcal{T}|^2. \tag{10}$$

and we obtain

$$|\mathcal{R}|^2 = 1 - \left(1 - \frac{m}{\omega}\Omega_{H,\alpha}\right)|\mathcal{T}|^2. \tag{11}$$

We observe that $\Omega_{H,\alpha}$ depends on the disclination, then, the same affects the quantity of removed energy of the hole.

Conclusins

- **The disclination modify the quantity of removed energy of the acoustic black hole.**

$$|\mathcal{R}|^2 = 1 - \left(1 - \frac{m}{\omega}\Omega_{H,\alpha}\right)|\mathcal{T}|^2 \, . \, ; \Omega_{H,\alpha} \equiv \frac{Bc}{\alpha A^2} \equiv \frac{\Omega}{\alpha}$$

- **It is possible to accentuate (**$\alpha > 0$ **such that** $\frac{m}{\omega}\Omega_{H,\alpha} > 1$ **or to attenuate (**$\alpha > 0$ **such that** $\frac{m}{\omega}\Omega_{H,\alpha} < 1$ **) the amplification of the removed.**

- **Exists the possibility to cancel the superradiance** $\left(1 - \frac{m}{\omega}\Omega_{H,\alpha}\right) = 0$ **to** α **equal to** $m\Omega/\omega$ **where** $\Omega \equiv \frac{Bc}{A^2}$ **is the angular velocity of the acoustic black hole in the absence of the disclination.**

Thank You!