Correction to the deviation of Mercury's periapsis from Newtonian theory : XIX Meeting of Physics

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Introduction

When planetary orbits are studied using Newton's formulation, it is shown that they describe closed orbits and does not adequately describe the deviation of Mercury's periapsis. That is why we seek the correction of the Mercury periapsis by adding a new thermium δU to the Newtonian potential.

Keppler orbits

The two-body problem [1] admits a complete general solution, to solve the problem we will first reduce the motion of the system in two motions (the center of mass and the motion of the particles with respect to the center of mass), starting from the interaction and considering the center of mass as the origin,

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - U(r) \tag{1}$$

where $m = m_1 m_2/(m_1 + m_2)$ reduced mass.

The impetus coincides with the angular momentum with respect to the CM, by which we apply the law of conservation of angular momentum that indicates the constancy of the air velocity; geometrically $df = \frac{1}{2}r.rd\phi$

$$M = mr^2\dot{\phi} \qquad\qquad M = 2m\dot{f}$$

Expressing $\dot{\phi}$ in terms of M and taking this value to energy, we obtain:

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{M^2}{mr^2} + U(r)$$

$$\dot{r} = \frac{dr}{dt} = \sqrt{\frac{2}{m}[E - U(r)] - \frac{M^2}{m^2r^2}}$$

clearing

$$t = \int \frac{dr}{\sqrt{\frac{2}{m}[E - U(r)] - \frac{M^2}{m^2r^2}}} + cte. \tag{2}$$

$$\triangle\phi = 2\int_{r_{min}}^{r_{max}} \frac{M dr/r^2}{\sqrt{2m[E - U(r)] - M^2/r^2}} \tag{3}$$

making $U(r) = -\alpha/r$ and solving the integral, then

$$\triangle\phi = 2\pi$$

Euler-Lagrange equations

Lagrangian of the system and the integral "*action*",

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)dt \tag{4}$$

It only depends on q and $\dot{q}$ since the mechanical state is completely defined by its coordinates and its speeds.
The minimum necessary condition of S is that the set of these terms vanishes, $q(\alpha_1) = q(\alpha_2) = 0$ is called the first variation of the integral, so

$$\delta S = \delta\int_{t_1}^{t_2} L(q, \dot{q}, t)dt = 0 \tag{5}$$

The time (t) does not depend on α and $d\dot{q}$ makes us uncomfortable, so

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \alpha}dt = \int_{t_1}^{t_2}\left(\frac{dq}{\partial q}\left(\frac{\partial L}{\partial \alpha}\right) + \frac{d}{dt}\left(\frac{dq}{d\alpha}\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial \dot{L}}{\partial \dot{q}}\left(\frac{dq}{d\alpha}\right)\right)dt$$

; there remains an integral that must be canceled for any δq, consequently

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

This can be generalized for "n" degrees of freedom.

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General relativity correction

General relativity already gives us a correction to the trajectory of our orbits.[2]

$$\Delta\phi = 2\pi + \delta\phi \tag{6}$$

$$\delta\phi = \frac{6\pi K m'}{c^2 a(1 - e^2)} \tag{7}$$

Correction to Newton's theory

We add to the Newton potential a correction δU , in the variation of (3)

$$\triangle\phi = -2\frac{\partial}{\partial M}\int_{r_{min}}^{r_{max}} \sqrt{2m(E - U) - M^2/r^2}dr$$

$$\triangle\phi = -2\frac{\partial}{\partial M}\int_{r_{min}}^{r_{max}} \sqrt{2m}\sqrt{E + \frac{\alpha}{r} - \frac{M^2}{2mr^2}}(1 - \frac{\delta U}{\sqrt{E + \frac{\alpha}{r} - \frac{M^2}{2mr^2}}})^{1/2}$$

the first term gives us 2 π and the second term we represent it in the form

$$\delta\phi \approx \frac{\sigma}{\partial M}\left(\frac{2m}{M}\right)\int_0^{\pi} r^2\delta U d\phi$$

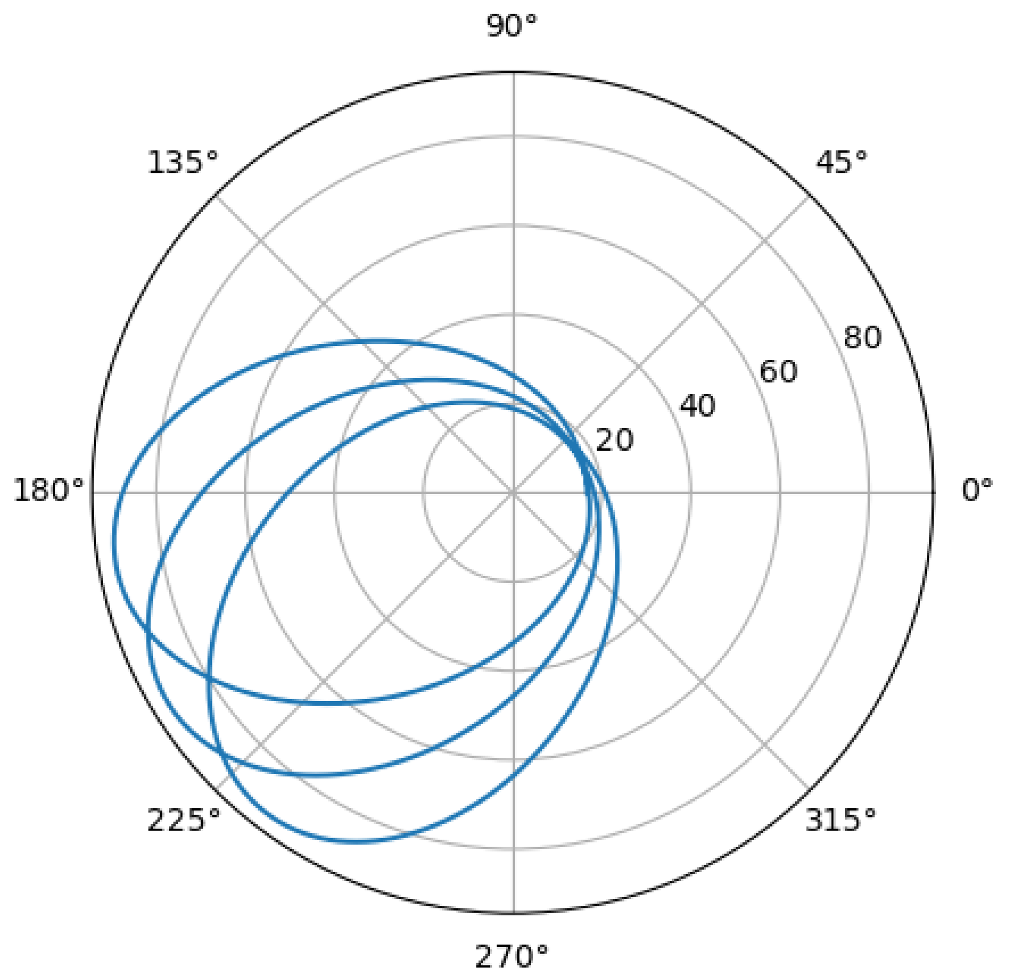
Landau proposes 2 fixes, $a)\delta U = \beta/r^2$ y
$b)r^2\delta U = \lambda/r$

$$\delta\phi \approx \frac{\partial}{\partial M}\int_0^{\pi}\frac{2m\lambda/r}{M}d\phi$$

$$\delta\phi \approx -\frac{6\pi\alpha\lambda m^2}{M^4} \approx -6\pi\lambda/\alpha p^2$$

Conclusion

Newton's formulation does not adequately describe the deviation of Mercury's periapsis, by which one recurs to a classical method starting from the Newtonian formalism. Without applying general relativity, which gives us the correction directly



References

[1] L. D. Landau and E. Lifshitz. *Física teórica. Mecánica*, volume 1. Reverte, 1970.
[2] L. D. Landau and E. M. Lifshitz. *Teoría clásica de campos*, volume 2. Reverté, 1992.