Modified non-linear Schrödinger and infinite towers of anomalous charges

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Abstract

Modifications of the non-linear Schrödinger model (MNLS) $i\partial_t\psi(x,t)+\partial_x^2\psi(x,t)-[\frac{\delta V}{\delta|\psi|^2}]\psi(x,t)=0$, where $\psi\in\mathbb{C}$ and $V:\mathbb{R}_+\to\mathbb{R}$, are considered. We show that the quasi-integrable MNLS models possess infinite towers of quasi-conservation laws for soliton-type configurations with a special complex conjugation, shifted parity and delayed time reversion ($\mathcal{CP}_s\mathcal{T}_d$) symmetry. Two infinite towers of exact non-local conservation laws are uncovered in the framework of the Riccati-type pseudo-potential approach.

Introduction

We search for additional quasi-conservation laws and study the role played by them in the phenomenon of quasi-integrability. A special complex conjugation, shifted parity and delayed time reversion ($\mathcal{CP}_s\mathcal{T}_d$) symmetry will play a central role in our constructions. An exact conservation law arises when a suitable linear combination of relevant quasi-conservation laws leads to the vanishing of the combined an omalies.

We numerically simulate various two-soliton configurations of the deformed model. The numerical simulations will allow us to verify the analytical expectations for the novel set of quasi-conservation laws, and check the behavior of some of the lowest order anomalies associated to the relevant new towers of anomalous conservation laws of the deformed model for a variety of values of the coupling constant and deformation parameters $\{\eta,\epsilon\}$.

The model

We will consider non-relativistic models in $(1+1)-$dimensions with equation of motion given by

$$i\frac{\partial}{\partial t}\psi(x,t)+\frac{\partial^2}{\partial x^2}\psi(x,t)-V^{(1)}(|\psi(x,t)|^2)\psi(x,t)\;=\;0, \tag{1}$$

$$V^{(n)}(I)\;\equiv\;\frac{d^n}{dI^n}V(I),\quad , \tag{2}$$

where $I\equiv\bar{\psi}\psi$ and ψ is a complex scalar field and the potential $V:\mathbb{R}_+\to\mathbb{R}$.

Modified NLS and anomalous charges

From eq. (1) and its complex conjugate one can write the quasi-conservation law

$$\partial_t[\frac{1}{2n}(\bar{\psi}\psi)^n]-\partial_x[\frac{1}{2n}i(\bar{\psi}\psi)^{n-1}(\bar{\psi}\partial_x\psi-\psi\partial_x\bar{\psi})]\;=\;\hat{\beta}_n, \tag{3}$$

$$\hat{\beta}_n\equiv-\frac{1}{2n}\partial_x[(\bar{\psi}\psi)^{n-1}]i(\bar{\psi}\partial_x\psi-\psi\partial_x\bar{\psi}),\quad ,n=1,2,3... \tag{4}$$

So, the towers of quasi-conservation laws, related to (3), follow as

$$\frac{d}{dt}\mathcal{Q}_n\;=\;\int dx\,\hat{\beta}_n;\quad n=2,3,... \tag{5}$$

$$\mathcal{Q}_n\;=\;\frac{1}{2n}\int dx\,(\bar{\psi}\psi)^n. \tag{6}$$

Therefore, for $\mathcal{CP}_s\mathcal{T}_d$ symmetric fields one must have the vanishing of the space-time integral of the anomaly $\hat{\beta}_n$, i.e.

$$\int_{-\widetilde{t}}^{\widetilde{t}}dt\int_{-\widetilde{x}}^{\widetilde{x}}dx\,\hat{\beta}_n=0,\quad\text{for }\;\widetilde{t}\to\infty,\;\;\widetilde{x}\to\infty,\quad n=2,3,... \tag{7}$$

So, the asymptotically conserved charges satisfy

$$\mathcal{Q}_n(t\to\infty)=\mathcal{Q}_n(t\to-\infty),\quad n=2,3,... \tag{8}$$

Numerical simulations

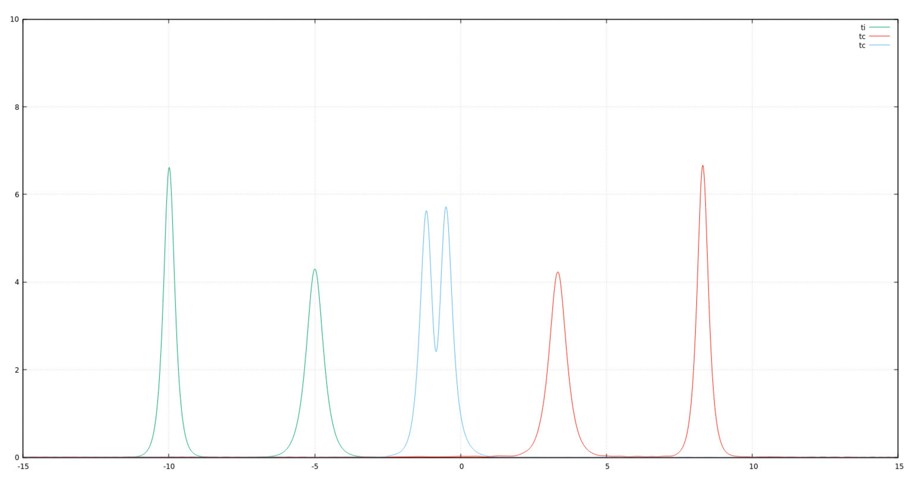

Figure 1: 2-bright solitons with amplitudes $|\psi_1|=6.83$, $|\psi_2|=4.446$ and velocities $v_1=12$, $v_2=6$, and $\epsilon=-0.06$, for initial (green), collision (blue) and final (red) succesive times.

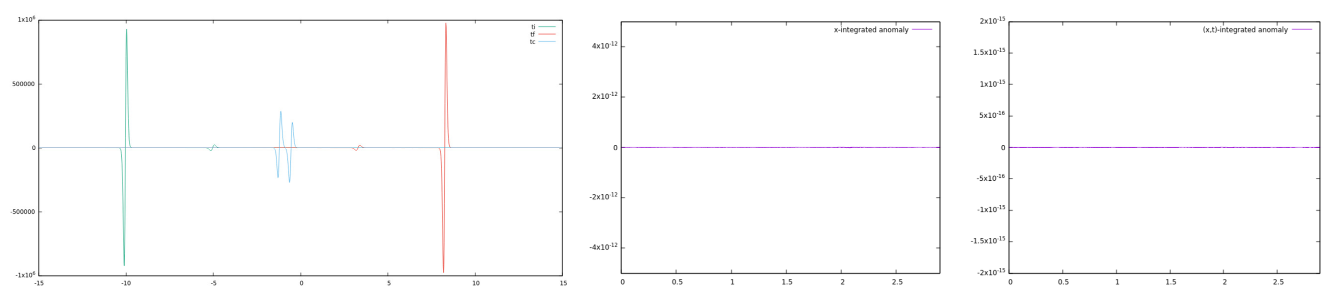

Figure 2: Left figure shows the profile of anomaly density $\hat{\beta}_2\;vs\;x$ for the 2-soliton collision of Fig. 11 for initial (green), collision (blue) and final (red) times. Middle figure shows the plot $\int\hat{\beta}_2dx\;vs\;t$ and the right shows the plot $\int dt\int dx\,\hat{\beta}_2\;vs\;t$.

Fig. 2 shows the simulations of the anomaly density $\hat{\beta}_2$ for the two soliton collision of Fig. 1. Left, it is plotted the anomaly density for three successive times, t_i, before collision (green), t_c, collision (blue), and, t_f, after collision (red) times, respectively. Middle, it is plotted the $x-$integration of the anomaly versus t; Right, it is plotted the $(x,t)-$integration of the anomaly versus t. The $(x,t)-$integration vanishes, with an error of order 10^{-16}.

Linear system formulation of modified AKNS system

Let us consider

$$A_x\;\equiv\;-\zeta\partial_x\bar{q}+2\bar{q}\left(\frac{a_0\zeta+\zeta^2a_1}{2\zeta\bar{q}+i\partial_x\bar{q}}\right), \tag{9}$$

$$A_t\;\equiv\;\zeta\int^x dx'\,\frac{b_0+b_1\zeta+b_2\zeta^2}{(2\zeta\bar{q}+i\partial_{x'}\bar{q})^2}, \tag{10}$$

$$\widetilde{A}_x\;\equiv\;\zeta\partial_xq+2q\left(\frac{\widetilde{a}_0\zeta+\zeta^2\widetilde{a}_1}{2\zeta q-i\partial_xq}\right), \tag{11}$$

$$\widetilde{A}_t\;\equiv\;-\zeta\int^x dx'\,\frac{\widetilde{b}_0+\widetilde{b}_1\zeta+\widetilde{b}_2\zeta^2}{(2\zeta q-i\partial_{x'}q)^2}, \tag{12}$$

$$\partial_x\begin{pmatrix}\Phi\\ \widetilde{\Phi}\end{pmatrix}=\mathcal{M}\begin{pmatrix}\Phi\\ \widetilde{\Phi}\end{pmatrix},\quad\mathcal{M}\equiv\begin{pmatrix}A_x&0\\0&\widetilde{A}_x\end{pmatrix} \tag{13}$$

$$\partial_t\begin{pmatrix}\Phi\\ \widetilde{\Phi}\end{pmatrix}=\mathcal{N}\begin{pmatrix}\Phi\\ \widetilde{\Phi}\end{pmatrix},\quad\mathcal{N}\equiv\begin{pmatrix}A_t&0\\0&\widetilde{A}_t\end{pmatrix}. \tag{14}$$

So, the compatibility condition of this system provides the zero-curvature eq.

$$\partial_t\mathcal{M}-\partial_x\mathcal{N}+\Big[\mathcal{M},\,\mathcal{N}\Big]=0. \tag{15}$$

The eq. (15) reproduces the MNLS eq. (1). Then, one can write the infinite tower of non-local conservation laws as

$$\partial_t[A_x\chi^{(1)}]-\partial_x[A_t\chi^{(1)}]\;=\;0, \tag{16}$$

$$\partial_t[A_x\chi^{(n)}]-\partial_x[A_t\chi^{(n)}]\;=\;0,\quad n=2,3,4,... \tag{17}$$

$$\partial_x\chi^{(n)}\;=\;A_x-A_x\chi^{(n-1)},\quad\partial_t\chi^{(n)}=A_t-A_t\chi^{(n-1)}. \tag{18}$$

Some conclusions and discussions

Quasi-integrability properties of certain deformations of the NLS model have been examined. New anomalous charges related to infinite towers of infinitely many quasi-conservation laws were uncovered. Moreover, an anomaly cancellation mechanism has been introduced in order to construct the exact conservation laws, since a convenient linear combination of a set of relevant anomalies identically vanishes. We have constructed two towers of infinite sets of non-local conservation laws associated to the linear formulations, respectively. These linear systems and their associated non-local charges deserve more careful considerations; in particular, regarding their relationships of their associated non-local currents with the so-called classical Yangians.

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