Interacting Modified Holographic Ricci Dark Energy Scenarios

Carlos Rodriguez-Benites [1] **Mauricio Cataldo** [2] **Marcial Vasquez** [3]

[1] Universidad Nacional de Barranca, Lima, Perú
[2] Grupo de Cosmología y Partículas Elementales, Universidad del Bío-Bío, Concepción, Chile
[3] Universidad Señor de Sipán, Chiclayo, Perú

Outline

Current content of the Universe

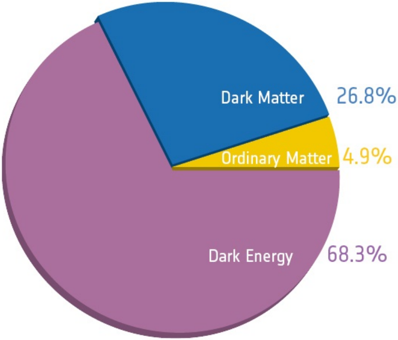

Figure: The content of the Universe, according to results from the Planck Satellite (2013). [arXiv:1303.5076v3]

General Relativity $\longleftrightarrow$ Symmetries: homogeneity and isotropy

$\Downarrow$

$$G^{\mu}_{\nu} = R^{\mu}_{\nu} - \frac{1}{2} g^{\mu}_{\nu} R = \kappa T^{\mu}_{\nu}$$

$\Downarrow$

relate

$\swarrow \quad \searrow$

Dynamics of the Universe $\longleftrightarrow$ Content of energy-momentum

General Relativity $\longleftrightarrow$ Symmetries: homogeneity and isotropy

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Dynamics of the Universe $\longleftrightarrow$ Content of energy-momentum

Holographic Dark Energy (HDE)

In this work our aim is to investigate a dark energy in the context of the holographic principle.

Holographic Principle

The number of degrees of freedom in a bounded system should be finite and is related to the area of its boundary.

Gerard 't Hooft[1], Leonard Susskind[2], and Jacob D. Bekenstein[3] .

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1 G. 't Hooft, "Dimensional Reduction In Quantum Gravity" in Salamfest 93, p. 284, gr-qc/9310026.

2 L. Susskind, J. Math. Phys. 36, 6377 (1995).

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In the literature, commonly the energy density of HDE is parametrized as $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$. In the holographic Ricci dark energy model[4], L is given by the average radius of the Ricci scalar curvature $|\mathcal{R}|^{-1/2}$, so in this case the density of the HDE is $\rho_x \propto \mathcal{R}$. In a spatially flat FLRW universe, the Ricci scalar of the spacetime is given by $|\mathcal{R}| = 6(\dot{H} + 2H^2)$, this model works fairly well in fitting the observational data, and it alleviates the cosmic coincidence problem[5].

Model

A generalization of the holographic Ricci dark energy model is proposed[6]

$$\rho_x = 3(\alpha H^2 + \beta \dot{H}) \tag{1}$$

where α and β are constants to be determined.

[4] S. del Campo, J. Fabris, R. Herrera, and W. Zimdahl. On holographic dark-energy models. 2011.

[5] C. Gao, F. Q. Wu, X. Chen and Y. G. Shen, Phys. Rev. D 79, 043511 (2009); Zhang, X. Phys. Rev. D 79, 103509 (2009); C. -J. Feng and X. Zhang, Phys. Lett. B 680 (2009) 399; T. F. Fu, J. F. Zhang, J. Q. Chen and X. Zhang, Eur.Phys. J. C 72, 1932 (2012).

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Spatially flat FLRW universe

In the framework of General Relativity and a homogeneous, isotropic and flat universe, the Friedmann-Lematre-Robertson-Walker (FLRW) metric

$$\boxed{ds^2 = dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]}\,, \tag{2}$$

where $a(t)$ is the scale factor and (t, r, θ, ϕ) are comoving coordinates. Then, from Einstein's Equation, we get

$$3\,H^2 = \rho\,, \tag{3}$$

$$2\,\dot{H} + 3\,H^2 = -p\,, \tag{4}$$

these are the so-called Friedmann equations. Also, the conservation of the energy-momentum tensor

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \dot{\rho} + 3H(\rho + p) = 0\,, \tag{5}$$

where ρ is the total energy density, p is the total pressure, and $8\pi G = c = 1$ is assumed. Also, $p = \omega\rho$.

HDE scenarios

No interaction

$$\left.\begin{array}{c} 3H^2 = \rho_1 + \rho_x \\ \dot{\rho}_1 + 3H\rho_1(1+\omega_1) = 0 \\ \dot{\rho}_x + 3H\rho_x(1+\omega) = 0 \\ \rho_x = 3(\alpha H^2 + \beta \dot{H}) \end{array}\right\} \longrightarrow \left\{\begin{array}{c} \text{with } \omega_1 \text{ constant} \\ \text{variables: } a,\ \rho_1, \rho_x \\ \Downarrow \\ \omega \text{ variable} \end{array}\right.$$

Interaction[7] [8]

$$\left.\begin{array}{c} 3H^2 = \rho_1 + \rho_x \\ \dot{\rho}_1 + 3H\rho_1(1+\omega_1) = -Q \\ \dot{\rho}_x + 3H\rho_x(1+\omega) = Q \\ \rho_x = 3(\alpha H^2 + \beta \dot{H}) \end{array}\right\} \longrightarrow \left\{\begin{array}{c} \text{with } \omega_1 \text{ constant} \\ \text{variables: } a,\ \rho_1, \rho_x \\ \text{given a } Q(\rho_1, \rho_x) \Rightarrow \omega \text{ variable} \\ \omega \text{ constant} \Rightarrow Q \text{ variable function} \end{array}\right.$$

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General analysis

We consider besides the Friedmann equation (3) and the conservation equation (5),

- the total density: $\rho = \rho_b + \rho_r + \rho_c + \rho_x$,
- the total pressure: $p = p_b + p_r + p_c + p_x$,
- dark sector: $\rho_d := \rho_c + \rho_x$,
- barotropic state equation: $p_i = \omega_i\, \rho_i$ with $\omega_b = 0$, $\omega_r = 1/3$, $\omega_c = 0$ and $\omega_x = \omega$.

We include a phenomenological interaction in the dark sector through

$$\rho_c' + \rho_c = -\Gamma \qquad \text{and} \qquad \rho_x' + (1+\omega)\,\rho_x = \Gamma\,. \tag{6}$$

where Γ is a function defining the interaction.

For the HDE (1) we obtain:

$$\boxed{\rho_x = \alpha\,\rho + \frac{3\beta}{2}\,\rho'}\,. \tag{7}$$

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In our scenario we have for baryons and radiation, respectively,

$$\rho_b = \rho_{b0}\, a^{-3} \quad \text{and} \quad \rho_r = \rho_{r0}\, a^{-4}\,. \tag{8}$$

The combining equations (6) - (8) we obtain

$$\boxed{\frac{3\beta}{2}\,\rho_d'' + \left(\alpha + \frac{3\beta}{2} - 1\right)\,\rho_d' + (\alpha - 1)\,\rho_d + \frac{1}{3}(2\beta - \alpha)\,\rho_{r0}\, a^{-4} = \Gamma} \tag{9}$$

The equation (9) can be easily solve when $\Gamma = \Gamma(\rho_d, \rho_d', \rho, \rho')$.

In our work we consider the following linear interactions[9] [10]:

$$\boxed{\Gamma_1 = \alpha_1\rho_c + \beta_1\rho_x, \quad \Gamma_2 = \alpha_2\rho_c' + \beta_2\rho_x' \quad \text{and} \quad \Gamma_3 = \alpha_3\rho_d + \beta_3\rho_d'.}$$

[9] F. Arevalo, A. Cid, and J. Moya, Eur. Phys. J. C77, 565 (2017)

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The energy density of the dark sector ρ_d

Notice that by rewriting equation (9) we get

$$\boxed{\rho_d'' + b_1\,\rho_d' + b_2\,\rho_d + b_3\,a^{-3} + b_4\,a^{-4} = 0}\,, \qquad (10)$$

where b_1, b_2, b_3, b_4 are parameters representing each interaction such that

	$\Gamma_1 = \alpha_1\,\rho_c + \beta_1\,\rho_x$	$\Gamma_2 = \alpha_2\,\rho_c' + \beta_2\,\rho_x'$	$\Gamma_3 = \alpha_3\,\rho_d + \beta_3\,\rho_d'$
b_1	$1+\alpha_1-\beta_1-\frac{2}{3\beta}(1-\alpha)$	$\frac{2\alpha-3\beta-2-2\alpha_2-2\alpha(\beta_2-\alpha_2)}{3\beta(1-\beta_2+\alpha_2)}$	$\frac{2}{3\beta}\left(\alpha+\frac{3\beta}{2}-1-\beta_3\right)$
b_2	$\frac{2}{3\beta}(\alpha(1-\beta_1+\alpha_1)-1-\alpha_1)$	$\frac{2(\alpha-1)}{3\beta(1-\beta_2+\alpha_2)}$	$\frac{2}{3\beta}(\alpha-1-\alpha_3)$
b_3	$(\beta_1-\alpha_1)\left(1-\frac{2\alpha}{3\beta}\right)\rho_{b0}$	$\frac{(2\alpha-3\beta)(\beta_2-\alpha_2)}{3\beta(1-\beta_2+\alpha_2)}\rho_{b0}$	0
b_4	$\frac{2}{3\beta}\left(\frac{1}{3}(2\beta-\alpha)-(\beta_1-\alpha_1)(\alpha-2\beta)\right)\rho_{r0}$	$\frac{2(2\beta-\alpha)-8(2\beta-\alpha)(\beta_2-\alpha_2)}{9\beta(1-\beta_2+\alpha_2)}\rho_{r0}$	$\frac{2}{9\beta}(2\beta-\alpha)\rho_{r0}$

The general solution of equation (10) has the form

$$\boxed{\rho_d(a) = A\,a^{-3} + B\,a^{-4} + C_1\,a^{3\,\lambda_1} + C_2\,a^{3\,\lambda_2}}\,, \tag{11}$$

where the integration constants are given by

$$\begin{aligned} C_1 &= \frac{-3A(1-\lambda_2) - B(4-3\lambda_2) - 9H_0^2\left((\lambda_2-1)\,\Omega_{c0} + (\lambda_2-\omega_0-1)\Omega_{x0}\right)}{3\,(\lambda_1-\lambda_2)}\,, \\ C_2 &= \frac{3A(1-\lambda_1) + B(4-3\lambda_1) + 9\,H_0^2\left((\lambda_1-1)\Omega_{c0} + (\lambda_1-\omega_0-1)\Omega_{x0}\right)}{3\,(\lambda_1-\lambda_2)}\,, \end{aligned} \tag{12}$$

and the coefficients in (11) are

$$A = \frac{b_3}{b_1 - b_2 - 1} \qquad \text{and} \qquad B = \frac{9b_4}{12b_1 - 9b_2 - 16}\,, \tag{13}$$

as well as

$$\lambda_{1,2} = -\frac{1}{2}\left(b_1 \pm \sqrt{b_1^2 - 4b_2}\right) \tag{14}$$

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The state parameter of the HDE

The state parameter of the HDE corresponds to the ratio $\omega = \frac{p_x}{\rho_x}$.

Using the expression (7) in equation (6), and the linear interactions Γ_i, we find

$$\boxed{\omega(a) = \frac{D_1\, a^{-3} + D_2\, a^{-4} + D_3\, a^{3\,\lambda_1} + D_4\, a^{3\lambda_2}}{\tilde{A}\, a^{-3} + \tilde{B}\, a^{-4} + \tilde{C}_1\, a^{3\,\lambda_1} + \tilde{C}_2\, a^{3\lambda_2}}}, \tag{15}$$

where $\tilde{A} = (2\alpha - 3\beta)(A + \rho_{b_0})$, $\tilde{B} = 2(\alpha - 2\beta)(B + \rho_{r_0})$ and $\tilde{C}_{1,2} = C_{1,2}(3\beta\lambda_{1,2} + 2\alpha)$.

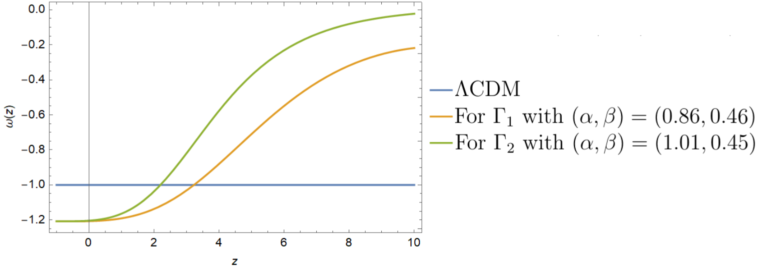
	$\Gamma_1 = \alpha_1\,\rho_c + \beta_1\,\rho_x$	$\Gamma_2 = \alpha_2\,\rho_c' + \beta_2\,\rho_x'$	$\Gamma_3 = \alpha_3\,\rho_d + \beta_3\,\rho_d'$
D_1	$2\alpha_1 A + (2\alpha - 3\beta)(\beta_1 - \alpha_1)(A + \rho_{b0})$	$-2\alpha_2 A + (3\beta - 2\alpha)(\beta_2 - \alpha_2)(A + \rho_{b0})$	$2(\alpha_3 + \beta_3)A$
D_2	$2\alpha_1 B + 2(\alpha - 2\beta)\left(\frac{1}{3} - \alpha_1 + \beta_1\right)(B + \rho_{r0})$	$-\frac{8}{3}\alpha_2 B + \frac{2}{3}(2\beta - \alpha)(-1 - \alpha_2 + \beta_2)(B + \rho_{r0})$	$2\left(\alpha_3 - \frac{4}{3}\beta_3\right)B + \frac{2}{3}(\alpha - 2\beta)(B + \rho_{r0})$
D_3	$C_1(2\alpha_1 + (2\alpha + 3\beta\lambda_1)(\beta_1 - \alpha_1 - 1 - \lambda_1))$	$C_1(2\alpha_2\lambda_1 - (2\alpha + 3\beta\lambda_1)(1 + \lambda_1(1 + \alpha_2 - \beta_2)))$	$C_1(2(\alpha_3 + \beta_3\lambda_1) - (2\alpha + 3\beta\lambda_1)(1 + \lambda_1))$
D_4	$C_2(2\alpha_1 + (2\alpha + 3\beta\lambda_2)(\beta_1 - \alpha_1 - 1 - \lambda_2))$	$C_2(2\alpha_2\lambda_2 - (2\alpha + 3\beta\lambda_2)(1 + \lambda_2(1 + \alpha_2 - \beta_2)))$	$C_2(2(\alpha_3 + \beta_3\lambda_2) - (2\alpha + 3\beta\lambda_2)(1 + \lambda_2))$

The state parameter of the HDE

Current values of the parameters[11]: $\Omega_{b0} = 0.0484$, $\Omega_{r0} = 1.25 \times 10^{-3}$, $\Omega_{c0} = 0.258$, $\Omega_{x0} = 0.692$, $H_0 = 67.8\, km\, s^{-1}\, Mpc^{-1}$, and $\omega_\Lambda = -1$.

For the linear interaction models, some reference values are[12]: $(\alpha_1, \beta_1) = (-0.0076, 0)$ and $(\alpha_2, \beta_2) = (0.0074, 0)$.

For the HDE scenario, some reference values are given[13 14]. Also, we consider $a = (1+z)^{-1}$.



[11] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A13 (2016).

[12] A. Cid, B. Santos, C. Pigozzo, T. Ferreira, J. Alcaniz. (2018), arXiv: 1805.02107[astro-ph.CO].

[13] S. Lepe and F. Peña, Eur. Phys. J. C 69, 575 (2010).

[14] F. Arévalo, P. Cifuentes, S. Lepe and F. Peña. Interacting Ricci-like holographic dark energy. (2014).

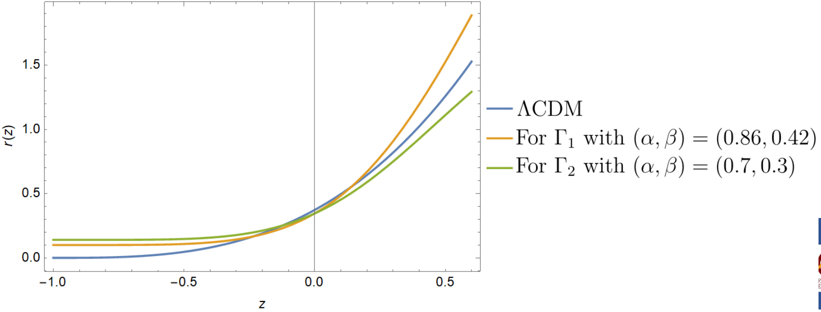
The coincidence parameter

To examine the problem of cosmological coincidence, we define $r \equiv \rho_c/\rho_x$.

In our work

$$r = \frac{\rho_d}{\left(\alpha - \frac{3\beta}{2}\right)\rho_b + (\alpha - 2\beta)\rho_r + \alpha\rho_d + \frac{3\beta}{2}\rho_d'} - 1\,. \tag{16}$$

We use (8) and (11) in the previous expression and obtain $r = r(a)$.
Then $r(a \to \infty) = \frac{2}{2\alpha+3\beta\lambda_i} - 1$, where $\lambda_i = \text{máx}\{\lambda_1, \lambda_2\}$ for $\lambda_i > 0$.

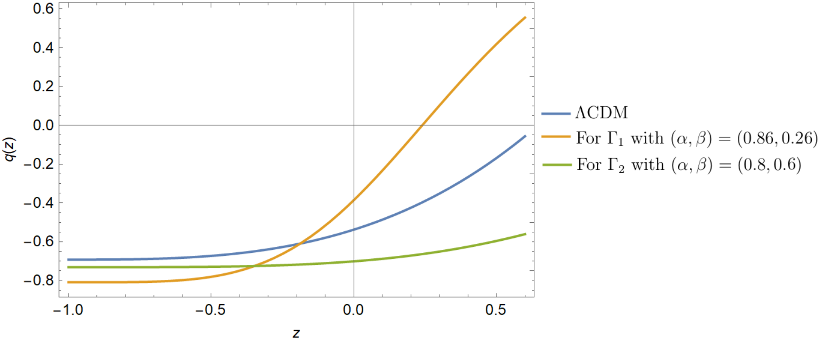


The deceleration parameter

The deceleration parameter q is a dimensionless measure of the cosmic acceleration in the evolution of the universe. It is defined by $q \equiv -\left(1+\frac{\dot{H}}{H^2}\right) = -\left(1+\frac{3\rho'}{2\rho}\right)$.

Using (11), we obtain

$$q(a) = -\left(1 + \frac{-3(\rho_{b0}+A)a^{-3} - 4(\rho_{r0}+B)a^{-4} + 3(C_1\lambda_1 a^{3\lambda_1} + C_2\lambda_2 a^{3\lambda_2})}{2(\rho_{b0}+A)a^{-3} + 2(\rho_{r0}+B)a^{-4} + 2(C_1 a^{3\lambda_1} + C_2 a^{3\lambda_2})}\right) \quad (17)$$



Conclusions and Perspectives.

- A theoretical model was developed according to the current components of the universe, such as baryons, radiation, dark dark cold and HDE, with interaction in the dark sector, obtaining for the HDE, the functions $\omega(z)$, $r(z)$ and $q(z)$.
- The proposed model was compared graphically ΛCDM, using the referential values for the HDE parameters and the given interactions.
- In the near future we expect to contrast the present scenarios with the observational data (SNe Ia, CC, BAO, CMB), using Bayesian statistics.
- We will also obtain the best fitting values for the model parameters and the use bayesian model selection criteria to compare these modells to ΛCDM.

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Conclusions and Perspectives.

- A theoretical model was developed according to the current components of the universe, such as baryons, radiation, dark dark cold and HDE, with interaction in the dark sector, obtaining for the HDE, the functions $\omega(z)$, $r(z)$ and $q(z)$.
- The proposed model was compared graphically ΛCDM, using the referential values for the HDE parameters and the given interactions.
- In the near future we expect to contrast the present scenarios with the observational data (SNe Ia, CC, BAO, CMB), using Bayesian statistics.
- We will also obtain the best fitting values for the model parameters and the use bayesian model selection criteria to compare these modells to ΛCDM.

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