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# Electric charge quantization in 331 models with exotic charges

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# The 331 model

- Straightforward extension of the SM of Elementary particles
- The gauge symmetry group is

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

- 3 Lepton families
- 3 quark families

# Charge assignment

The general relation for the electric charge operator in a 331 model

$$Q = \alpha T_3 + \beta T_8 + \gamma X$$

where  $\alpha = 1$  (to properly set the  $W$  boson electric charge in the model)

the upper sign corresponds to the fundamental representation of the Gell–Mann matrices and the lower sign to the conjugate representation.

$$Q = \begin{pmatrix} \pm \frac{1}{2} \left( 1 + \frac{\beta}{\sqrt{3}} \right) & 0 & 0 \\ 0 & \frac{1}{2} (\mp 1 \pm \beta/\sqrt{3}) & 0 \\ 0 & 0 & \mp \frac{1}{3} \beta \sqrt{3} \end{pmatrix} + \gamma X I_{3 \times 3} \dots (E1)$$

# Scalar sector

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

The scalar sector of the model is given by:

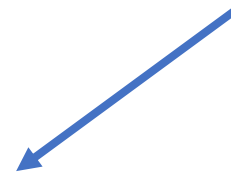
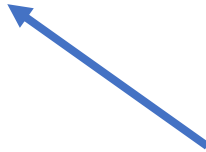
$$\langle \eta^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix} \sim (1, 3, X_\eta)$$

Three scalar triplets

$$\langle \rho^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\rho \\ 0 \\ 0 \end{pmatrix} \sim (1, 3, X_\rho)$$

**Hypercharge**

$$\langle \chi^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix} \sim (1, 3, X_\chi)$$



# Scalar sector

**the charge operator must annihilate the vacuum** (to keep the conservation of the electric charge)

$$Q\langle\eta^0\rangle = 0 \quad \rightarrow \quad \gamma = -\frac{1}{2}\left(1 + \frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\eta}$$

$$Q\langle\rho^0\rangle = 0 \quad \rightarrow \quad \gamma = -\frac{1}{2}\left(-1 + \frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\rho}$$

$$Q\langle\chi^0\rangle = 0 \quad \rightarrow \quad \gamma = \frac{\beta}{\sqrt{3}}\frac{1}{X_\chi}$$

$$X_\eta + X_\rho + X_\chi = 0$$

# Scalar sector

For  $\beta = \frac{1}{3\sqrt{3}}$

$$\begin{aligned}\gamma &= \frac{1}{9X_\eta} \\ X_\eta &= -5X_\chi \\ X_\rho &= 4X_\chi\end{aligned}$$

And the in the equation (E1):

$$Q = \text{diag}(\pm 5/9, \mp 4/9, \mp 1/9) + \frac{X}{9X_\chi} I_{3 \times 3}$$

Target  
Hypercharge



# Leptonic sector

The leptonic sector includes

$$\psi_{i_L} = (\nu_i, e_i^-, E_i)_L^T \sim (1, 3, X_{\ell_i}) \longrightarrow \text{Triplets}$$

No right-handed  
neutrinos

$$e_{i_R}^- \sim (1, 1, X_{e_i})$$

$$E_{i_R} \sim (1, 1, X_{E_i})$$

Singlets

$$i = 1, 2, 3$$

# Quark sector

The two first quark families forms  $SU(3)_L$  antitriplets

$$Q_{aL} = (d_a, -u_a, D_a)_L^T \sim (3, 3^*, X_{Q_a})$$

$$u_{aR} \sim (3, 1, X_{u_a})$$

$$d_{aR} \sim (3, 1, X_{d_a})$$

$$D_{aR} \sim (3, 1, X_{D_a})$$

$$a = 1, 2$$



# Quark sector

The third family is assigned to  $SU(3)_L$  triplets

$$Q_{3L} = (t, b, T)_L^T \sim (3, 3, X_{Q_3})$$

$$t_R \sim (3, 1, X_t)$$

$$b_R \sim (3, 1, X_b)$$

$$T_R \sim (3, 1, X_T)$$

# The Yukawa lagrangian

## Quark sector

$$\begin{aligned} -\mathcal{L}_Y^{\text{quarks}} &= f_{ab}^u \overline{Q_{aL}} \rho^* u_{bR} + f_{ab}^d \overline{Q_{aL}} \eta^* d_{bR} + f_{ab}^D \overline{Q_{aL}} \chi^* D_{bR} \\ &+ f^b \overline{Q_{3L}} \rho b_R + f^t \overline{Q_{3L}} \eta t_R + f^T \overline{Q_{3L}} \chi T_R + \text{h. c.} \end{aligned}$$

## Leptonic sector

$$-\mathcal{L}_Y^{\text{leptons}} = F_{ij}^e \overline{\psi_{iL}} \rho e_{jR} + F_{ij}^E \overline{\psi_{iL}} \chi E_{jR} + \text{h. c.}$$

# Hypercharge

Constraints from replicas between families

$$X_{Q_1} = X_{Q_2} \equiv X_Q$$

$$X_{l_1} = X_{l_2} = X_{l_3} \equiv X_l$$

$$X_{u_1} = X_{u_2} = X_t \equiv X_u$$

$$X_{d_1} = X_{d_2} = X_b \equiv X_d$$

$$X_{e_1} = X_{e_2} = X_{e_3} \equiv X_e$$

$$X_{E_1} = X_{E_2} = X_{E_3} \equiv X_E$$

$$X_{D_1} = X_{D_2} \equiv X_D$$

# Constraints from $U(1)_X$ invariance

From  $U(1)_X$  invariance of the Yukawa Lagrangian

$$X_Q = X_d - X_\eta$$

$$X_Q = X_d - X_\chi$$

$$X_Q = X_u - X_\rho$$

$$X_{Q_3} = X_\eta + X_t$$

$$X_{Q_3} = X_\chi + X_T$$

$$X_{Q_3} = X_\rho + X_b$$

$$X_e = X_\ell - X_\rho$$

$$X_E = X_\ell - X_\chi$$

$$\overline{Q_{1L}} \rho'^* u'_{1R} = \overline{Q_{1L}} e^{-X_Q \rho'^*} e^{-X_\rho} u_{1R} e^{X_u}$$



$U(1)_X$  gauge transformation

# Constraints from the cancellation of anomalies

The quantum restrictions arising from the cancellation of anomalies imply

$$[SU(3)_C]^2 U(1)_X \rightarrow A_C = 3 \sum_q X_{qL} - \sum_q X_{qR} = 0,$$

$$[SU(3)_L]^2 U(1)_X \rightarrow A_L = 3 \sum_q X_{qL} + \sum_\ell X_{\ell L} = 0,$$

$$[\text{Grav}]^2 U(1)_X \rightarrow A_G = 3 \sum_{\ell,q} [X_{\ell L} + 3X_{qL}] \\ - \sum_{\ell,q} [X_{\ell R} + 3X_{qR}] = 0,$$

$$[U(1)_X]^3 \rightarrow A_X = 3 \sum_{\ell,q} [X_{\ell L}^3 + 3X_{qL}^3] \\ - \sum_{\ell,q} [X_{\ell R}^3 + 3X_{qR}^3] = 0,$$

# Constraints from the cancellation of anomalies

In our case:

$$A_C = 3\{2X_Q + X_{Q_3}\} - (2X_u + 2X_d + 2X_D + X_t + X_b + X_T) = 0$$

$$A_L = 3\{2X_Q + X_{Q_3}\} + 3X_\ell = 0$$

$$A_G = 9X_\ell - 3X_e - 3X_E = 0$$

# Results

For  $\beta = \frac{1}{3\sqrt{3}}$ , we obtain

$$X_\ell = -5 X_\chi$$

$$X_Q = 2X_\chi$$

$$X_{Q_3} = X_\chi$$

$$X_D = 3X_\chi$$

$$X_T = 0$$

$$X_d = -3X_\chi$$

$$X_u = 6X_\chi$$

$$X_e = -9X_\chi$$

$$X_E = -6X_\chi$$

# Results

Using:

$$Q = \text{diag}(\pm 5/9, \mp 4/9, \mp 1/9) + \frac{X}{9X_\chi} I_{3 \times 3}$$

For example:

$$Q\psi_L = \left[ \text{diag} \left( \frac{5}{9}, -\frac{4}{9}, -\frac{1}{9} \right) + \frac{X_\ell}{9X_\chi} I_{3 \times 3} \right] \psi_L, \quad \text{but } X_\ell = -5 X_\chi$$

$$Q\psi_L = \left[ \text{diag} \left( \frac{5}{9}, -\frac{4}{9}, -\frac{1}{9} \right) - \frac{5}{9} I_{3 \times 3} \right] \psi_L$$

$$Q\psi_L = \text{diag} \left( 0, -1, -\frac{2}{3} \right) \psi_L$$



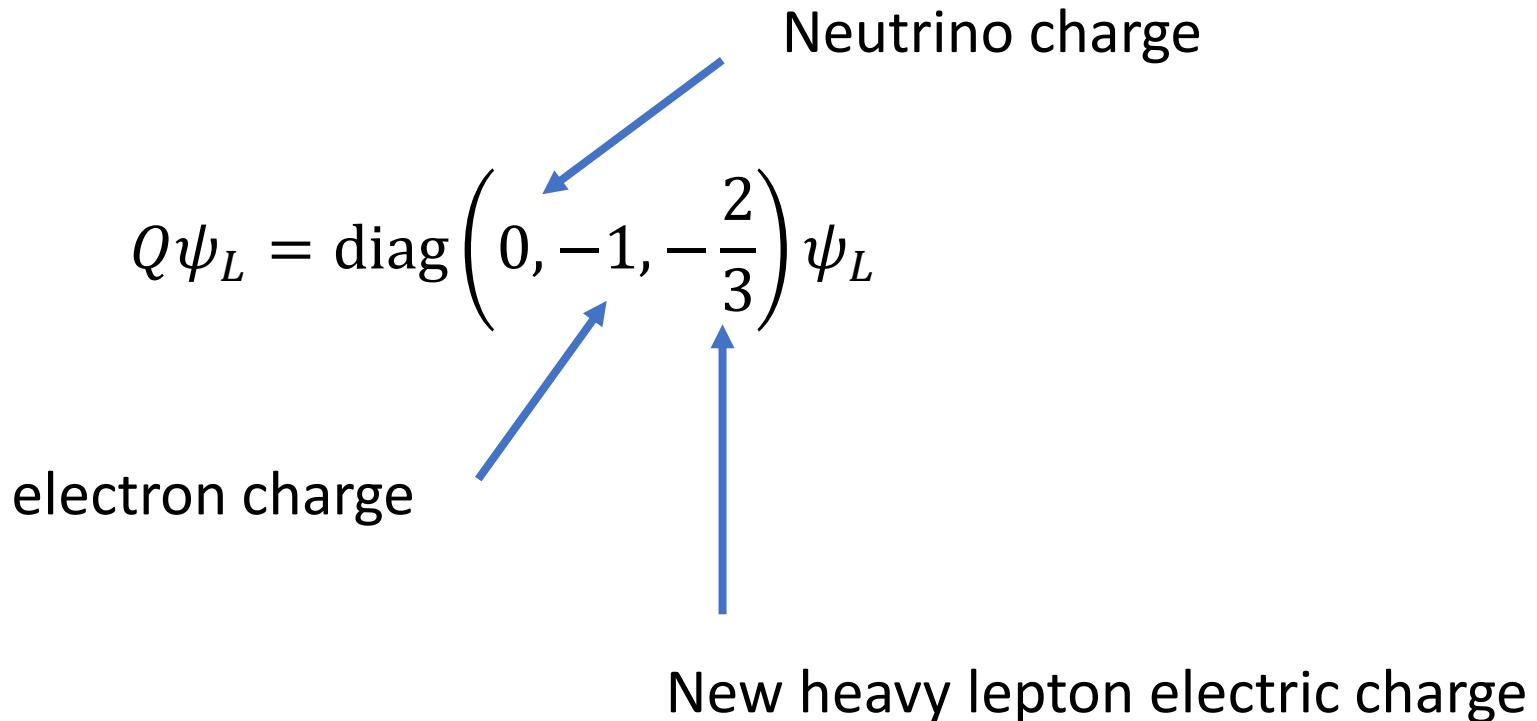
# Results

$$Q\psi_L = \text{diag} \left( 0, -1, -\frac{2}{3} \right) \psi_L$$

Neutrino charge

electron charge

New heavy lepton electric charge



# Results

$$Q_{\nu e, \mu, \tau} = 0, \quad Q_{e, \mu, \tau} = -1, \quad Q_{E, M, T} = -2/3.$$

$$Q_{d, s} = -1/3, \quad Q_{u, c} = 2/3, \quad Q_{D_1, D_2} = 1/3$$

$$Q_b = -1/3, \quad Q_t = 2/3, \quad Q_T = 0.$$

# Results

$$Q_{\nu_{e,\mu,\tau}} = 0, \quad Q_{e,\mu,\tau} = -1, \quad Q_{E,M,T} = -2/3.$$

**New leptons  
charge**

$$Q_{d,s} = -1/3, \quad Q_{u,c} = 2/3, \quad Q_{D_1,D_2} = 1/3$$

**New quarks  
charge**

$$Q_b = -1/3, \quad Q_t = 2/3, \quad Q_T = 0.$$

# Conclusions

- In this work, we have considered one version of the 331 model, with the particular feature of containing extra leptons with fractional electric charges and non-standard electric charges for the new quarks.
- By considering constraints from the classical and quantum levels, we have shown, that the quantization of the electric charge can be obtained by using the Yukawa sector and the cancellation of chiral anomalies when the three families are taken together and independent of the neutrino, as happens in the other 331 versions.
- As can be observed from our procedure, different  $\beta$  values produce different constraint equations as a result of imposing the  $U(1)_X$  invariance of the Yukawa Lagrangian and the cancellation of anomalies. This is because  $\beta$  fixes the fermion representations in the multiplets of the group.
- We think that the extension of the charge quantization for an arbitrary  $\beta$  is not straightforward.