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Electric charge quantization in 331 models with exotic charges

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The 331 model

- Straighforward extensión of the SM of Elementary particles

- The gauge symmetry group is

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

- 3 Lepton families
- 3 quark families

Charge assignment

The general relation for the electric charge operator in a 331 model

$$Q = \alpha T_3 + \beta T_8 + \gamma X$$

where $\alpha = 1$ (to properly set the W boson electric charge in the model)

the upper sign corresponds to the fundamental representation of the Gell–Mann matrices and the lower sign to the conjugate representation.

$$Q = \begin{pmatrix} \pm\frac{1}{2}\left(1+\frac{\beta}{\sqrt{3}}\right) & 0 & 0 \\ 0 & \frac{1}{2}(\mp 1 \pm \beta/\sqrt{3}) & 0 \\ 0 & 0 & \mp\frac{1}{3}\beta\sqrt{3} \end{pmatrix} + \gamma X I_{3\times 3} \ldots (E1)$$

Scalar sector

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X$$

The scalar sector of the model is given by:

Three scalar triplets

$$\langle \eta^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix} \sim (1,3,X_\eta)$$

$$\langle \rho^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\rho \\ 0 \\ 0 \end{pmatrix} \sim (1,3,X_\rho)$$

$$\langle \chi^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix} \sim (1,3,X_\chi)$$

Hypercharge

Scalar sector

the charge operator must annihilate the vacuum (to keep the conservation of the electric charge)

$$Q\langle\eta^0\rangle = 0 \quad \rightarrow \quad \gamma = -\frac{1}{2}\left(1 + \frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\eta}$$

$$Q\langle\rho^0\rangle = 0 \quad \rightarrow \quad \gamma = -\frac{1}{2}\left(-1 + \frac{\beta}{\sqrt{3}}\right)\frac{1}{X_\rho}$$

$$Q\langle\chi^0\rangle = 0 \quad \rightarrow \quad \gamma = \frac{\beta}{\sqrt{3}}\frac{1}{X_\chi}$$

$$X_\eta + X_\rho + X_\chi = 0$$

Scalar sector

For $\boldsymbol{\beta = \frac{1}{3\sqrt{3}}}$

$$\gamma = \frac{1}{9X_\eta}$$
$$X_\eta = -5X_\chi$$
$$X_\rho = 4X_\chi$$

Target Hypercharge

And the in the equation $(E1)$:

$$Q = \mathrm{diag}(\pm 5/9, \mp 4/9, \mp 1/9) + \frac{X}{9X_\chi}\mathrm{I}_{3\times 3}$$

Leptonic sector

The leptonic sector includes

$$\psi_{i_L} = (\nu_i, e_i^-, E_i)_L^T \sim (1,3,X_{\ell_i}) \longrightarrow \text{Triplets}$$

No right-handed neutrinos

$$e_{i_R}^- \sim (1,1,X_{e_i})$$

$$E_{i_R} \sim (1,1,X_{E_i})$$

Singlets

$$i = 1,2,3$$

Quark sector

The two first quark families forms $SU(3)_L$ antitriplets

$$Q_{a_L} = (d_a, -u_a, D_a)_L^T \sim (3, 3^*, X_{Q_a})$$

$$u_{a_R} \sim (3,1, X_{u_a})$$

$$d_{a_R} \sim (3,1, X_{d_a})$$

$$D_{a_R} \sim (3,1, X_{D_a})$$

$$a = 1,2$$

Quark sector

The third family is assigned to $SU(3)_L$ triplets

$$Q_{3_L} = (t, b, T\ )_L^T \sim (3,3, X_{Q_3})$$

$$t_R \sim (3,1, X_t)$$

$$b_R \sim (3,1, X_b)$$

$$T_R \sim (3,1, X_T)$$

The Yukawa lagrangian

Quark sector

$$-\mathcal{L}_Y^{\text{quarks}} = f^u_{ab}\,\overline{Q_{aL}}\,\rho^* u_{bR} + f^d_{ab}\,\overline{Q_{aL}}\,\eta^* d_{bR} + f^D_{ab}\,\overline{Q_{aL}}\,\chi^* D_{bR}$$

$$+ f^b\,\overline{Q_{3L}}\,\rho\, b_R + f^t\,\overline{Q_{3L}}\,\eta\, t_R + f^T\,\overline{Q_{3L}}\,\chi\, T_R + \text{h.c.}$$

Leptonic sector

$$-\mathcal{L}_Y^{\text{leptons}} = F^e_{ij}\,\overline{\psi_{iL}}\,\rho\, e_{jR} + F^E_{ij}\,\overline{\psi_{iL}}\,\chi\, E_{jR} + \text{h.c.}$$

Hypercharge

Constraints from replicas between families

$$X_{Q_1} = X_{Q_2} \equiv X_Q$$
$$X_{\ell_1} = X_{\ell_2} = X_{\ell_3} \equiv X_\ell$$
$$X_{u_1} = X_{u_2} = X_t \equiv X_u$$
$$X_{d_1} = X_{d_2} = X_b \equiv X_d$$
$$X_{e_1} = X_{e_2} = X_{e_3} \equiv X_e$$
$$X_{E_1} = X_{E_2} = X_{E_3} \equiv X_E$$
$$X_{D_1} = X_{D_2} \equiv X_D$$

Constraints from $\boldsymbol{U(1)_X}$ invariance

From $U(1)_X$ invariance of the Yukawa Lagrangian

$$X_Q = X_d - X_\eta$$
$$X_Q = X_d - X_\chi$$
$$X_Q = X_u - X_\rho$$

$$X_{Q_3} = X_\eta + X_t$$
$$X_{Q_3} = X_\chi + X_T$$
$$X_{Q_3} = X_\rho + X_b$$

$$X_e = X_\ell - X_\rho$$
$$X_E = X_\ell - X_\chi$$

$$\boxed{\overline{Q_{1L}}\,'\rho^{*\prime}u'_{1R} = \overline{Q_{1L}}\;e^{-X_Q}\rho^*\;e^{-X_\rho}\,u_{1R}\,e^{X_u}}$$

↓

$U(1)_X$ gauge transformation

Constraints from the cancellation of anomalies

The quantum restrictions arising from the cancellation of anomalies imply

$$[SU(3)_C]^2 U(1)_X \rightarrow A_C = 3\sum_q X_{q_L} - \sum_q X_{q_R} = 0,$$

$$[SU(3)_L]^2 U(1)_X \rightarrow A_L = 3\sum_q X_{q_L} + \sum_\ell X_{\ell_L} = 0,$$

$$[\mathrm{Grav}]^2 U(1)_X \rightarrow A_\mathrm{G} = 3\sum_{\ell,q} [X_{\ell_L} + 3X_{q_L}] - \sum_{\ell,q} [X_{\ell_R} + 3X_{q_R}] = 0,$$

$$[U(1)_X]^3 \rightarrow A_X = 3\sum_{\ell,q} \left[X^3_{\ell_L} + 3X^3_{q_L}\right] - \sum_{\ell,q} \left[X^3_{\ell_R} + 3X^3_{q_R}\right] = 0,$$

Constraints from the cancellation of anomalies

In our case:

$$A_C = 3\{2X_Q + X_{Q_3}\} - (2X_u + 2X_d + 2X_D + X_t + X_b + X_T) = 0$$

$$A_L = 3\{2X_Q + X_{Q_3}\} + 3X_\ell = 0$$

$$A_G = 9X_\ell - 3X_e - 3X_E = 0$$

Results

For $\boldsymbol{\beta = \frac{1}{3\sqrt{3}}}$, we obtain

$$X_\ell = -5\,X_\chi$$
$$X_Q = 2X_\chi$$
$$X_{Q_3} = X_\chi$$
$$X_D = 3X_\chi$$
$$X_T = 0$$
$$X_d = -3X_\chi$$
$$X_u = 6X_\chi$$
$$X_e = -9X_\chi$$
$$X_E = -6X_\chi$$

Results

Using:

$$Q = \mathrm{diag}(\pm 5/9, \mp 4/9, \mp 1/9) + \frac{X}{9X_\chi} \mathrm{I}_{3\times 3}$$

For example:

$$Q\psi_L = \left[\mathrm{diag}\left(\frac{5}{9}, -\frac{4}{9}, -\frac{1}{9}\right) + \frac{X_\ell}{9X_\chi} \mathrm{I}_{3\times 3}\right] \psi_L, \quad \text{but } X_\ell = -5\, X_\chi$$

$$Q\psi_L = \left[\mathrm{diag}\left(\frac{5}{9}, -\frac{4}{9}, -\frac{1}{9}\right) - \frac{5}{9} \mathrm{I}_{3\times 3}\right] \psi_L$$

$$Q\psi_L = \mathrm{diag}\left(0, -1, -\frac{2}{3}\right) \psi_L$$

Results

Neutrino charge

$$Q\psi_L = \mathrm{diag}\left(0, -1, -\frac{2}{3}\right)\psi_L$$

electron charge

New heavy lepton electric charge

Results

$$\mathcal{Q}_{\nu_{e,\mu,\tau}} = 0, \quad \mathcal{Q}_{e,\mu,\tau} = -1, \quad \mathcal{Q}_{E,M,\mathcal{T}} = -2/3.$$

$$\mathcal{Q}_{d,s} = -1/3, \quad \mathcal{Q}_{u,c} = 2/3, \quad \mathcal{Q}_{D_1,D_2} = 1/3$$

$$\mathcal{Q}_b = -1/3, \quad \mathcal{Q}_t = 2/3, \quad \mathcal{Q}_T = 0.$$

Results

$$\mathcal{Q}_{\nu_{e,\mu,\tau}} = 0, \quad \mathcal{Q}_{e,\mu,\tau} = -1, \quad \mathcal{Q}_{E,M,\mathcal{T}} = -2/3.$$

New leptons charge

$$\mathcal{Q}_{d,s} = -1/3, \quad \mathcal{Q}_{u,c} = 2/3, \quad \mathcal{Q}_{D_1,D_2} = 1/3$$

$$\mathcal{Q}_{b} = -1/3, \quad \mathcal{Q}_{t} = 2/3, \quad \mathcal{Q}_{T} = 0.$$

New quarks charge

Conclusions

- In this work, we have considered one version of the 331 model, with the particular feature of containing extra leptons with fractional electric charges and non-standard electric charges for the new quarks.
- By considering constraints from the classical and quantum levels, we have shown, that the quantization of the electric charge can be obtained by using the Yukawa sector and the cancellation of chiral anomalies when the three families are taken together and independent of the neutrino, as happens in the other 331 versions.
- As can be observed from our procedure, different β values produce different constraint equations as a result of imposing the $U(1)_X$ invariance of the Yukawa Lagrangian and the cancellation of anomalies. This is because β fixes the fermion representations in the multiplets of the group.
- We think that the extension of the charge quantization for an arbitrary β is not straightforward.