# Traversable wormholes, linearized perturbations of BTZ metrics and ANEC violation

J. Bonifacio, C. Rivera, T. Vargas

Updated: 2020/09/24

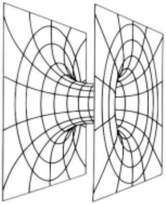
Grupo de Física Teórica, Universidad Nacional Mayor de San Marcos

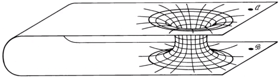
# Traversable Wormholes in Classical Gravity & $AdS/CFT$

Traversable Wormholes (TW) in **Classical Gravity** requires a violation of ANEC (produced by some *exotic matter*, etc):

$$\boxed{\int_{-\infty}^{+\infty} T_{\mu\nu} k^{\mu} k^{\nu} d\lambda \geq 0}$$

ANEC (Averaged Null Energy Condition)





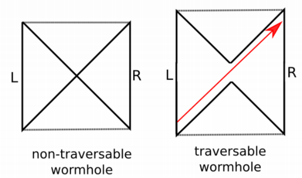
There are several causal inconsistencies in this picture: *Closed timelike curves, warp drives, time machines...*

But in **Holography** ... A specific toy model in $AdS_3/CFT_2$ (modified by a small double trace deformation) produces an amount of negative energy density in the backreacted geometry, explicitly violating ANEC without having the causal inconsistencies described above. (GFW'16)

# Linearized Perturbations of BTZ Black Holes

Turning on a coupling between the L/R boundaries of a BTZ blackhole,

$$\delta H(t_1) = -\int d^{d-1}x_1\, h(t_1, x_1)\mathcal{O}_R(t_1, x_1)\mathcal{O}_L(-t_1, x_1)$$



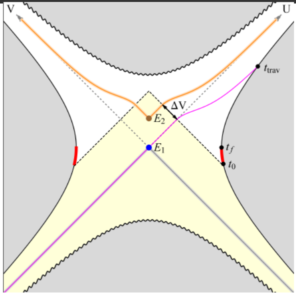

produces a backreaction in the bulk geometry (BTZ) due by a small spherically symmetric perturbation $h_{\mu\nu}$. In Kruskal coordinates (U,V) :

$$ds^2 = h_{UU}dU^2 + 2\left(-\frac{2l^2}{(1+UV)^2} + h_{UV}\right)dUdV + 2h_{U\phi}dUd\phi$$
$$+h_{VV}dV^2 + 2h_{V\phi}dVd\phi + \left(\frac{r_+^2(1-UV)^2}{(1+UV)^2} + h_{\phi\phi}\right)d\phi^2$$

The backreacted geometry is expressed as $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$, modifying the Einstein equations at a linealized level in the UU component:

$$\frac{1}{2}\left[l^{-2}(h_{UU} + \partial_U(Uh_{UU})) - r_+^{-2}\partial_U^2 h_{\phi\phi}\right] = 8\pi G_N \langle T_{UU}\rangle$$

# Violation of the Averaged Null Energy Condition (ANEC)




$$8\pi G_N \int T_{UU} dU = \frac{1}{2} l^{-2} \int h_{UU} dU$$

The opening of the "throat" is $\Delta V$:

$$\Delta V(U) = -(2g_{UV}(V=0))^{-1} \int_{-\infty}^{U} h_{UU} dU$$

Since $g_{UV}(V=0) < 0$ and $\Delta V(U) < 0$, the integral $\int_{-\infty}^{U} h_{UU} dU$ needs to be negative in order to have a traversable wormhole.

From the linearized perturbed Einstein equations, this requirement is no other that an explicit violation of the ANEC condition,

$$\boxed{\int T_{UU} dU \sim \int_{-\infty}^{U} h_{UU} dU \quad \rightarrow \int T_{UU} dU < 0}$$