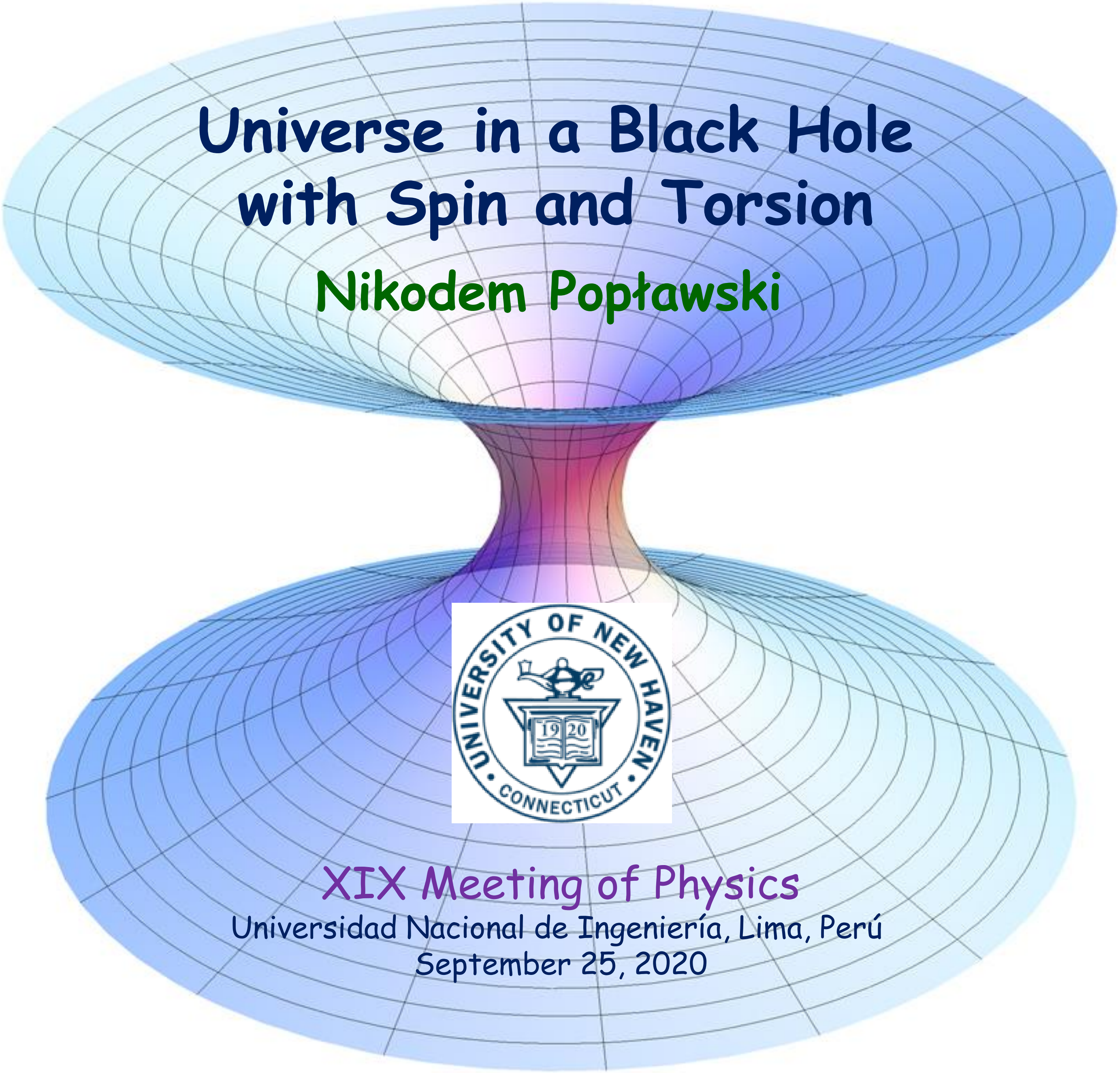

Universe in a Black Hole with Spin and Torsion

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XIX Meeting of Physics

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Universo en un Agujero Negro con Espín y Torsión

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XIX Encuentro de Física

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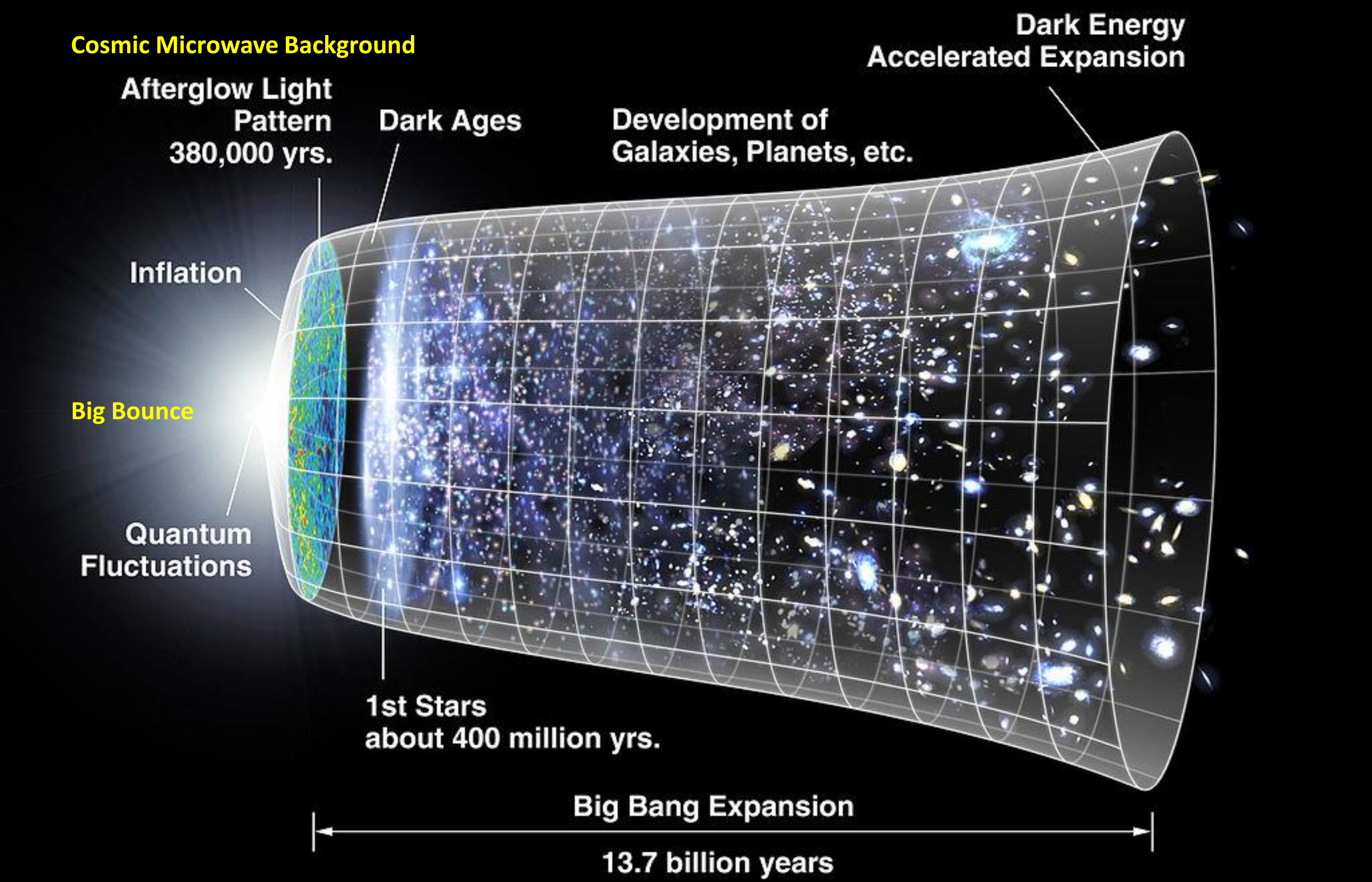

Cosmic Microwave Background
Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Big Bounce
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years

Problemas de la relatividad general

La cosmología se basa en la relatividad general (RG), que describe la gravedad como la curvatura del espacio-tiempo.

- Singularidades: puntos con densidad de materia infinita.
- Incompatible con la mecánica cuántica. La gravedad cuántica puede resolver el problema de la singularidad.
- Las ecuaciones de campo contienen la conservación del momento angular orbital, lo que contradice la ecuación de Dirac que proporciona la conservación del momento angular total (orbital + espín) y permite el intercambio espín-órbita en mecánica cuántica.

Extensión más simple de RG para incluir el espín: **Teoría de Einstein-Cartan**. Puede resolver el problema de la singularidad.

Problemas de la cosmología

- Singularidad del Big-Bang.

- ¿Qué causó el Big Bang? ¿Qué existía antes?

- La inflación (expansión exponencial del Universo temprano) resuelve los problemas de planitud y horizonte, y predice el espectro observado de perturbaciones de CMB. ¿Qué causó la inflación? (se suelen utilizar campos escalares hipotéticos)

- ¿Cómo terminó la inflación? (sin inflación eterna)

La teoría de Einstein-Cartan reemplaza el Big Bang por un no singular **Gran Rebote** (Big Bounce). La dinámica después del rebote puede explicar los problemas de planitud y horizonte.

Gravedad de Einstein-Cartan-Sciama-Kibble

- El **tensor de torsión** es una variable además de la métrica. Es la parte antisimétrica de la conexión afín.

$$S^k{}_{ij} = \Gamma^{\;k}_{[i\,j]}$$

- La derivada covariante de la métrica desaparece (como en RG), y la conexión tiene una parte puramente métrica y una parte de torsión.
- La densidad Lagrangiana es proporcional al escalar de curvatura de Ricci R (como en RG), construido a partir de la conexión. El tensor de curvatura se puede descomponer en una puramente métrica parte y parte que contiene la torsión y sus derivadas.

arXiv.org > gr-qc > arXiv:0911.0334

Hehl, von der Heyde, Kerlick & Nester, RMP 48, 393 (1976)

Gravedad de Einstein-Cartan-Sciama-Kibble

- La variación de la acción total de la gravedad y la materia con respecto a la torsión da las ecuaciones de campo de Cartan:

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2}\kappa s_{ikj}$$

La **torsión** es proporcional a la densidad de **espín** de los fermiones. EC difiere significativamente del RG en densidades > 10^{45} kg/m^3. En vacío, se reduce a RG. EC pasa todas las pruebas que hace RG.

- La variación con respecto a la métrica da las ecuaciones de Einstein: la **curvatura** es proporcional a la densidad de **energía y momento**. Usando la descomposición por curvatura, se pueden escribir como RG con el tensor de energía-momento con términos cuadráticos en densidad de espín.

$$G^{ik} = \kappa T^{ik} + \frac{1}{2}\kappa^2\left(s^{ij}{}_j s^{kl}{}_l - s^{ij}{}_l s^{kl}{}_j - s^{ijl} s^k{}_{jl} + \frac{1}{2}s^{jli} s_{jl}{}^k + \frac{1}{4}g^{ik}(2s_j{}^l{}_m s^{jm}{}_l - 2s_j{}^l{}_l s^{jm}{}_m + s^{jlm} s_{jlm})\right)$$

La teoría de Einstein-Cartan también puede tener consecuencias en la física de partículas, que pueden probarse experimentalmente.
PLB 690, 73 (2010); FOP 50, 900 (2020)
Por tanto, brinda posibilidades de colaboración conmigo y con diferentes grupos de investigación.

Visitando Machu Picchu, 23 de noviembre de 2018, antes del XII Simposio Latinoamericano de Física de Altas Energías, Pontificia Universidad Católica del Perú, Lima, Perú.

ECSK gravity and spinors

ECSK gravity removes the GR constraint that the affine connection $\Gamma_{i\,j}^{\,k}$ be symmetric by regarding **the antisymmetric part of the connection, the torsion tensor** $S^k_{\ ij} = \Gamma^{\ \ k}_{[i\,j]}$, as a variable. Varying the total Lagrangian density $-\frac{1}{2\kappa}R\sqrt{-g} + L_\mathrm{m}$, where R is the Ricci scalar and L_m is the Lagrangian density of matter, with respect to the contortion tensor $C_{ijk} = S_{ijk} + S_{jki} + S_{kji}$ gives the Cartan equations

$$S^j_{\ ik} - S_i\delta^j_k + S_k\delta^j_i = -\frac{1}{2}\kappa s_{ik}^{\ \ j},$$

where $S_i = S^k_{\ ik}$ and $s^{ijk} = 2(\delta L_\mathrm{m}/\delta C_{ijk})/\sqrt{-g}$ is the spin tensor.

The Dirac Lagrangian density for a free spinor ψ with mass m, minimally coupled to the gravitational field, is given by $L_\mathrm{m} = \frac{i}{2}\sqrt{-g}(\bar{\psi}\gamma^i\psi_{;i} - \bar{\psi}_{;i}\gamma^i\psi) - m\sqrt{-g}\bar{\psi}\psi$, where γ^i are the Dirac matrices obeying $\gamma^{(i}\gamma^{k)} = g^{ik}I$, a semicolon denotes a covariant derivative with respect to the connection, and a colon denotes a Riemannian covariant derivative with respect to the Christoffel symbols:

$$\psi_{;k} = \psi_{:k} + \frac{1}{4}C_{ijk}\gamma^{[i}\gamma^{j]}\psi.$$

The resulting spin tensor (and thus torsion) is quadratic in spinor fields:

$$s^{ijk} = -e^{ijkl}s_l, \quad s^i = \frac{1}{2}\bar{\psi}\gamma^i\gamma^5\psi.$$

ECSK gravity and spin fluid

At macroscopic scales, Dirac fields can be averaged and described as a spin fluid:

$$s_{ij}{}^{k} = s_{ij}u^k, \quad s_{ij}u^j = 0.$$

The terms in the effective energy-momentum tensor that are quadratic in the spin tensor do not vanish after averaging:

$$G^{ij} = \kappa\Big(\epsilon - \frac{1}{4}\kappa s^2\Big)u^i u^j - \kappa\Big(p - \frac{1}{4}\kappa s^2\Big)(g^{ij} - u^i u^j),$$

where

$$s^2 = \frac{1}{2}s_{ij}s^{ij} > 0 \propto n_{\rm f}^2$$

is the averaged square of the spin density.

The Einstein–Cartan equations for a spin fluid are therefore equivalent to the GR Einstein equations for a perfect fluid with:

$$\tilde{\epsilon} = \epsilon - \alpha n_{\rm f}^2, \quad \tilde{p} = p - \alpha n_{\rm f}^2,$$

where ϵ and p are the thermodynamic energy density and pressure, $n_{\rm f}$ is the number density of fermions, and $\alpha = \kappa(\hbar c)^2/32$ with $\kappa = 8\pi G/c^4$.

Gravitational collapse of fluid sphere

A spherically symmetric gravitational field is given by the Tolman metric:

$$ds^2 = e^{\nu(\tau,R)}c^2 d\tau^2 - e^{\lambda(\tau,R)}dR^2 - e^{\mu(\tau,R)}(d\theta^2 + \sin^2\theta\, d\phi^2),$$

where ν, λ, and μ are functions of a time coordinate τ and a radial coordinate R. Coordinate transformations $\tau \to \tilde{\tau}(\tau)$ and $R \to \tilde{R}(R)$ do not change the form of the metric.

The components of the Einstein tensor corresponding to this metric that do not vanish identically are:

$$G_0^0 = -e^{-\lambda}\Big(\mu'' + \frac{3\mu'^2}{4} - \frac{\mu'\lambda'}{2}\Big) + \frac{e^{-\nu}}{2}\Big(\dot{\lambda}\dot{\mu} + \frac{\dot{\mu}^2}{2}\Big) + e^{-\mu},$$

$$G_1^1 = -\frac{e^{-\lambda}}{2}\Big(\frac{\mu'^2}{2} + \mu'\nu'\Big) + e^{-\nu}\Big(\ddot{\mu} - \frac{\dot{\mu}\dot{\nu}}{2} + \frac{3\dot{\mu}^2}{4}\Big) + e^{-\mu},$$

$$G_2^2 = G_3^3 = -\frac{e^{-\nu}}{4}(\dot{\lambda}\dot{\nu} + \dot{\mu}\dot{\nu} - \dot{\lambda}\dot{\mu} - 2\ddot{\lambda} - \dot{\lambda}^2 - 2\ddot{\mu} - \dot{\mu}^2)$$

$$-\frac{e^{-\lambda}}{4}(2\nu'' + \nu'^2 + 2\mu'' + \mu'^2 - \mu'\lambda' - \nu'\lambda' + \mu'\nu'),$$

$$G_0^1 = \frac{e^{-\lambda}}{2}(2\dot{\mu}' + \dot{\mu}\mu' - \dot{\lambda}\mu' - \dot{\mu}\nu'),$$

where a dot denotes differentiation with respect to $c\tau$ and a prime denotes differentiation with respect to R.

Gravitational collapse of fluid sphere

In the comoving frame of reference, the spatial components of the four-velocity u^μ vanish. The nonzero components of the energy–momentum tensor for a spin fluid, $T_{\mu\nu} = (\tilde{\epsilon} + \tilde{p})u_\mu u_\nu - \tilde{p} g_{\mu\nu}$, are: $T^0_0 = \tilde{\epsilon}$, $T^1_1 = T^2_2 = T^3_3 = -\tilde{p}$. The Einstein field equations $G^\mu_\nu = \kappa T^\mu_\nu$ in this frame of reference are:

$$G^0_0 = \kappa\tilde{\epsilon}, \quad G^1_1 = G^2_2 = G^3_3 = -\kappa\tilde{p}, \quad G^1_0 = 0.$$

The covariant conservation of the energy–momentum tensor gives

$$\dot{\lambda} + 2\dot{\mu} = -\frac{2\dot{\tilde{\epsilon}}}{\tilde{\epsilon} + \tilde{p}}, \;\; \nu' = -\frac{2\tilde{p}'}{\tilde{\epsilon} + \tilde{p}},$$

where the constants of integration depend on the allowed transformations $\tau \to \tilde{\tau}$ and $R \to \tilde{R}$.

If the pressure is homogeneous (no pressure gradients), then $\tilde{p}' = 0$, which gives $\nu' = 0$. Therefore, $\nu = \nu(\tau)$ and a transformation $\tau \to \tilde{\tau}$ can bring ν to zero and $g_{00} = e^\nu$ to 1. The system of coordinates becomes synchronous. Defining $r(\tau, R) = e^{\mu/2}$ turns the metric into

$$ds^2 = c^2 d\tau^2 - e^{\lambda(\tau,R)} dR^2 - r^2(\tau, R)(d\theta^2 + \sin^2\theta\, d\phi^2).$$

Every particle in a collapsing fluid sphere is represented by a radial coordinate R that ranges from 0 (at the center) to R_0 (at the surface).

Gravitational collapse of fluid sphere

The Einstein field equations reduce to

$$\begin{aligned}
\kappa\tilde{\epsilon} &= -\frac{e^{-\lambda}}{r^2}(2rr'' + r'^2 - rr'\lambda') + \frac{1}{r^2}(r\dot{r}\dot{\lambda} + \dot{r}^2 + 1),\\
-\kappa\tilde{p} &= \frac{1}{r^2}(-e^{-\lambda}r'^2 + 2r\ddot{r} + \dot{r}^2 + 1),\\
-2\kappa\tilde{p} &= -\frac{e^{-\lambda}}{r}(2r'' - r'\lambda') + \frac{\dot{r}\dot{\lambda}}{r} + \ddot{\lambda} + \frac{1}{2}\dot{\lambda}^2 + \frac{2\ddot{r}}{r},\\
2\dot{r}' - \dot{\lambda}r' &= 0.
\end{aligned}$$

Integrating the last equation gives

$$e^{\lambda} = \frac{r'^2}{1 + f(R)}, \tag{1}$$

where f is a function of R satisfying a condition $1 + f > 0$ (Landau & Lifshitz, *The Classical Theory of Fields*). Substituting this relation into the second field equation gives $2r\ddot{r} + \dot{r}^2 - f = -\kappa\tilde{p}r^2$, which is integrated to

$$\dot{r}^2 = f(R) + \frac{F(R)}{r} - \frac{\kappa}{r}\int \tilde{p}r^2 dr,$$

where F is a positive function of R.

Gravitational collapse of fluid sphere

Substituting the last two equations into the first field equation gives a relation $\kappa(\tilde{\epsilon}+\tilde{p}) = F'(R)/(r^2 r')$, leading to

$$\dot{r}^2 = f(R) + \frac{\kappa}{r}\int_0^R \tilde{\epsilon} r^2 r' dR. \tag{2}$$

If the mass of the sphere is M, then the Schwarzschild radius $r_g = 2GM/c^2$ of the black hole that forms from the sphere is equal to

$$r_g = \kappa \int_0^{R_0} \tilde{\epsilon} r^2 r' dR.$$

These two quations give $\dot{r}^2(\tau, R_0) = f(R_0) + r_g/r(\tau, R_0)$. If $r_0 = r(0, R_0)$ is the initial radius of the sphere and the sphere is initially at rest, then $\dot{r}(0, R_0) = 0$. Consequently, the value of R_0 is determined by

$$f(R_0) = -\frac{r_g}{r_0}.$$

Substituting $r = e^{\mu/2}$ and (1) into the first conservation law gives the first law of thermodynamics for the effective energy density and pressure:

$$\frac{d}{d\tau}(\tilde{\epsilon} r^2 r') + \tilde{p}\frac{d}{d\tau}(r^2 r') = 0. \tag{3}$$

Collapse of spin fluid sphere

If we assume that the spin fluid is composed of an ultrarelativistic matter in kinetic equilibrium, then $\epsilon = h_\star T^4$, $p = \epsilon/3$, and $n_{\rm f} = h_{n\rm f} T^3$, where T is the temperature of the fluid, $h_\star = (\pi^2/30)(g_{\rm b} + (7/8)g_{\rm f})k^4/(\hbar c)^3$, and $h_{n\rm f} = (\zeta(3)/\pi^2)(3/4)g_{\rm f}k^3/(\hbar c)^3$. For standard-model particles, $g_{\rm b} = 29$ and $g_{\rm f} = 90$. The effective energy density and pressure are thus:

$$\tilde{\epsilon} = h_\star T^4 - \alpha h_{n\rm f}^2 T^6, \quad \tilde{p} = \frac{1}{3}h_\star T^4 - \alpha h_{n\rm f}^2 T^6.$$

Since the pressure has no gradient, the temperature only depends on τ, and so does the energy density. This scenario describes a homogeneous sphere. The first law of thermodynamics (3) gives

$$r^2 r' T^3 = g(R), \tag{4}$$

where g is a function of R. Putting this relation into (2) gives

$$\dot{r}^2 = f(R) + \frac{\kappa}{r}(h_\star T^4 - \alpha h_{n\rm f}^2 T^6)\int_0^R r^2 r' dR. \tag{5}$$

Equations (4) and (5) give the function $r(\tau, R)$, which with (1) gives $\lambda(\tau, R)$. The integration of (5) also contains the initial value $\tau_0(R)$. The metric therefore depends on three arbitrary functions: $f(R)$, $g(R)$, and $\tau_0(R)$.

N. Popławski, arXiv:2008.02136.

Collapse of spin fluid sphere

We seek a solution of (4) and (5) as

$$f(R) = -\sin^2 R, \quad r(\tau, R) = a(\tau)\sin R, \tag{6}$$

where $a(\tau)$ is a nonnegative function of τ. For this choice of functions, (4) gives $a^3T^3\sin^2 R\cos R = g(R)$, in which separation of the variables τ and R leads to

$$g(R) = \mathrm{const}\cdot\sin^2 R\cos R, \quad a^3T^3 = \mathrm{const}.$$

Consequently,

$$aT = a_0T_0, \quad \frac{\dot{T}}{T} + \frac{H}{c} = 0, \tag{7}$$

where $a_0 = a(\tau = 0)$ and $T_0 = T(\tau = 0)$ are the values at the initial time, and $H = c\dot{a}/a$. Substituting (6) into (5) gives

$$\dot{a}^2 + 1 = \frac{\kappa}{3}(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2, \tag{8}$$

which has a form of the Friedmann equation for the scale factor a as a function of the cosmic time τ in a closed, homogeneous universe. The quantity H is the Hubble parameter of this universe. Using (7) in (8) yields

$$\dot{a}^2 = -1 + \frac{\kappa}{3}\Big(\frac{h_\star T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4}\Big). \tag{9}$$

Collapse of spin fluid sphere

The relations (6) determine the constants:

$$\sin R_0 = \left(\frac{r_g}{r_0}\right)^{1/2}, \quad a(0) = \left(\frac{r_0^3}{r_g}\right)^{1/2}.$$

Substituting the initial values $a(0)$ and $\dot{a}(0) = 0$ into (8), in which the second term on the right-hand side is negligible, gives $Mc^2 = (4\pi/3)r_0^3 h_\star T_0^4$. This relation indicates the equivalence of mass and energy of a fluid sphere with radius r_0 and determines T_0. An event horizon for the entire sphere forms when $r(\tau, R_0) = r_g$, which is equivalent to $a = (r_g r_0)^{1/2}$. Equation (9) has two turning points, $\dot{a} = 0$, if

$$\frac{r_0^3}{r_g} > \frac{3\pi G\hbar^4 h_{nf}^4}{8h_\star^3} \sim l_{\rm Planck}^2,$$

which is satisfied for astrophysical systems that form black holes.

Substituting (6) into (1) gives $e^{\lambda(\tau,R)} = a^2$. Consequently, the square of an infinitesimal interval in the interior of a collapsing spin fluid is given by

$$ds^2 = c^2 d\tau^2 - a^2(\tau)dR^2 - a^2(\tau)\sin^2 R(d\theta^2 + \sin^2\theta\, d\phi^2).$$

This metric has a form of the closed Friedmann–Lemaître–Robertson–Walker metric and describes a part of a **closed universe** with $0 \le R \le R_0$ (like dust).

Nonsingular bounce

Equation (9) can be solved analytically in terms of an elliptic integral of the second kind, giving the function $a(\tau)$ and then $r(\tau, R) = a(\tau)\sin R$:

$$\dot{a}^2 = -1 + \frac{\kappa}{3}\Big(\frac{h_\star T_0^4 a_0^4}{a^2} - \frac{\alpha h_{nf}^2 T_0^6 a_0^6}{a^4}\Big). \qquad (9)$$

The **value of a never reaches zero** because as a decreases, the right-hand side of (9) becomes negative, contradicting the left-hand side. The change of the sign occurs when $a < (r_g r_0)^{1/2}$, that is, after the event horizon forms. Consequently, all particles with $R > 0$ fall within the event horizon but never reach $r = 0$ (the only particle at the center is the particle that is initially at the center, with $R = 0$). **A singularity is replaced with a nonsingular bounce.** Nonzero values of a give finite values of T and thus finite values of ϵ, p, and n_{f}.

After the bounce, the matter expands on the other side of the event horizon as a **new universe**. This universe has a closed geometry (constant positive curvature). The quantity $a(\tau)$ is the scale factor. The universe is oscillatory: the value of a oscillates between the two turning points. The value of R_0 does not change. A turning point at which $\ddot{a} > 0$ is a bounce, and a turning point at which $\ddot{a} < 0$ is a crunch. This universe has an infinite number of identical cycles.

N. Popławski, Astrophys. J. 832, 96 (2016).
G. Unger and N. Popławski, Astrophys. J. 870, 78 (2019).

Nonsingular bounce

The Raychaudhuri equation for a congruence of geodesics without four-acceleration and rotation is $d\theta/ds = -\theta^2/3 - 2\sigma^2 - R_{\mu\nu}u^\mu u^\nu$, where θ is the expansion scalar, σ^2 is the shear scalar, and $R_{\mu\nu}$ is the Ricci tensor. For a spin fluid, the last term in this equation is equal to $-\kappa(\tilde{\epsilon}+3\tilde{p})/2$. Consequently, the necessary and sufficient condition for avoiding a singularity in a black hole is $-\kappa(\tilde{\epsilon}+3\tilde{p})/2 > 2\sigma^2$. For a relativistic spin fluid, $p=\epsilon/3$, this condition is equivalent to

$$2\kappa\alpha n_{\mathrm{f}}^2 > 2\sigma^2 + \kappa\epsilon. \tag{10}$$

Without torsion, the left-hand side of (10) would be absent and this inequality could not be satisfied, resulting in a singularity. **Torsion provides a necessary condition for preventing a singularity.** In the absence of shear, this condition is also sufficient.
(Hehl, Trautman, Kopczyński)

The presence of shear opposes the effects of torsion. The shear scalar σ^2 grows with decreasing a like $\sim a^{-6}$, which is the same power law as that for n_{f}^2. Therefore, if the initial shear term dominates over the initial torsion term in (10), then it will dominate at later times during contraction and a singularity will form. To avoid a singularity if the shear is present, n_{f}^2 must grow faster than $\sim a^{-6}$. Consequently, **fermions must be produced** in a black hole during contraction.

Nonsingular bounce

The production rate of particles in a contracting or expanding universe can be phenomenologically given by

$$\frac{1}{c\sqrt{-g}}\frac{d(\sqrt{-g}n_{\rm f})}{dt} = \frac{\beta H^4}{c^4}, \tag{11}$$

where $g = -a^6 \sin^4 R \sin^2\theta$ is the determinant of the metric tensor and β is a nondimensional production rate. With particle production, the second equation in (7) turns into

$$\frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{n{\rm f}} T^3} - 1\Big). \tag{12}$$

Particle production changes the power law $n_{\rm f}(a)$:

$$n_{\rm f} \sim a^{-(3+\delta)},$$

where δ varies with τ. Putting this relation into (11) gives

$$\delta \sim -a^{\delta}\dot{a}^3.$$

During contraction, $\dot{a} < 0$ and thus $\delta > 0$. The term $n_{\rm f}^2 \sim a^{-6-2\delta}$ grows faster than $\sigma^2 \sim a^{-6}$ and a singularity is avoided. **Particle production and torsion together reverse the effects of shear, generating a bounce.**

The universe in a black hole

The dynamics of the nonsingular, relativistic universe in a black hole is described by equations (8) and (12):

$$\dot{a}^2 + 1 = \frac{\kappa}{3}(h_\star T^4 - \alpha h_{nf}^2 T^6)a^2, \quad \frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{nf} T^3} - 1\Big),$$

where $H = c\dot{a}/a$. These equations, with the initial conditions $a(0) = (r_0^3/r_g)^{1/2}$ and $\dot{a}(0) = 0$, give the functions $a(\tau)$ and $T(\tau)$.

The shear would enter the right-hand side of the first equation as an additional positive term that is proportional to a^{-4}. When the universe becomes nonrelativistic, the term $h_\star T^4$ changes into a positive term proportional to a^{-1}. The cosmological constant enters as a positive term proportional to a^2.

Particle production increases the maximum size of the scale factor that is reached at a crunch. Consequently, a new cycle is larger and lasts longer then the previous cycle. R_0 is given by $\sin^3 R_0 = r_g/a(0)$, where $a(0)$ is the maximum scale factor in the first cycle. Since the maximum scale factor in the next cycle is larger, the value of $\sin R_0$ decreases. As cycles proceed, R_0 approaches π (the value for a completely closed universe).

A parent black hole creating a new, baby universe becomes an **Einstein–Rosen bridge** (unidirectional wormhole) to that universe.

Closed Universe

If the Universe is closed, it is analogous to the 2-dimensional surface of a 3-dimensional sphere. The Universe would be mathematically the 3-dimensional hypersurface of a 4-dimensional hypersphere.

The 3-dimensional space in which the balloon expands is not analogous to any higher dimensional space. Points off the surface of the balloon are not in the Universe in this analogy.

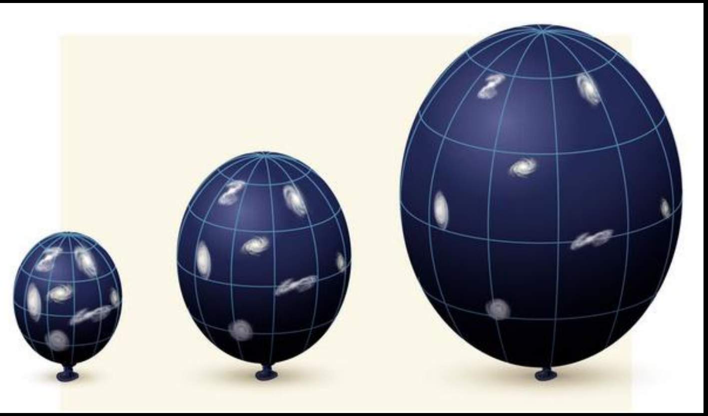


Image credit: One-Minute Astronomer

The balloon radius = scale factor a.

The Universe expands (Hubble law).

The Universe may be finite (closed) or infinite (flat or open).

Inflation

During expansion ($H > 0$), if β is too big, then the right-hand side could become positive:

$$\frac{\dot{T}}{T} = \frac{H}{c}\Big(\frac{\beta H^3}{3c^3 h_{n\mathrm{f}} T^3} - 1\Big).$$

In this case, the temperature would grow with increasing a, which would lead to eternal inflation. Consequently, there is an upper limit to the production rate: the maximum of the function $(\beta H^3)/(3c^3 h_{n\mathrm{f}} T^3)$ must be lesser than 1.

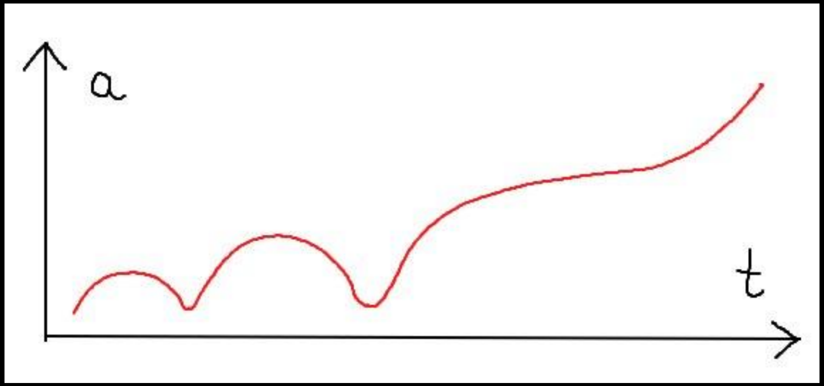
If $(\beta H^3)/(3c^3 h_{n\mathrm{f}} T^3)$ increases after a bounce to a value that is slightly lesser than 1, then T would become approximately constant. Accordingly, H would be also nearly constant and the scale factor a would grow exponentially, generating inflation. Since the energy density would be also nearly constant, the universe would produce enormous amounts of matter and entropy. Such an expansion would last until the right-hand side of drops below 1. Consequently, inflation would last a finite period of time. After this period, the effects of torsion weaken and the universe smoothly enters the radiation-dominated expansion, followed by the matter-dominated expansion.

Torsion and particle production can generate **finite inflation** without scalar fields and reheating.

N. J. Popławski, Phys. Lett. B 694, 181 (2010).
N. Popławski, Astrophys. J. 832, 96 (2016).

Dark energy ceasing oscillations

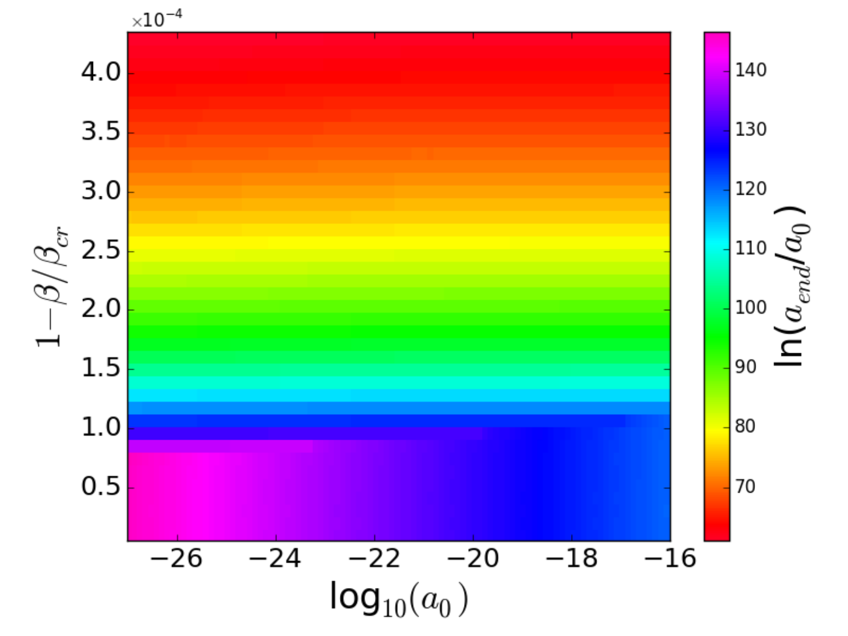
If quantum effects in the gravitational field near a bounce do not produce enough matter, then the closed Universe reaches the maximum size and then contracts to another bounce, beginning the new cycle. Because of matter production, a new cycle reaches larger size and last longer than the previous cycle.



When the Universe reaches a size at which the cosmological constant is dominating, then it avoids another contraction and starts expanding to infinity.

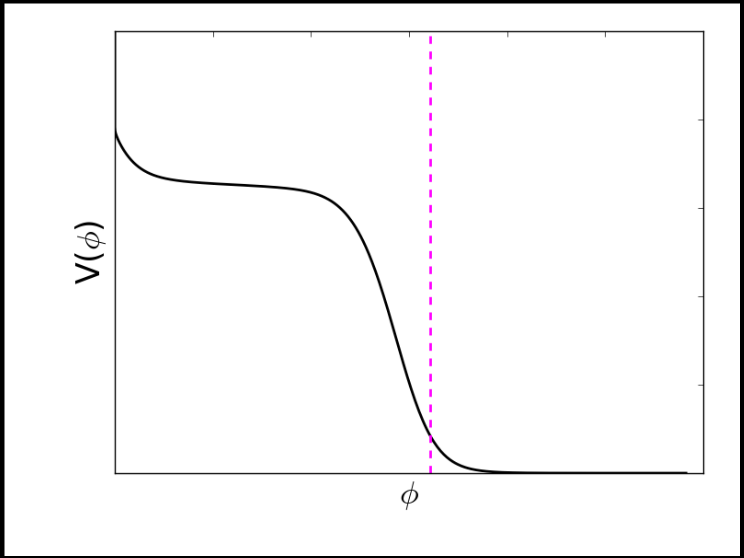
- The numbers of e-folds and bounces (until the Universe reaches the radiation-matter equality) depend on the particle production but are not too sensitive to the initial scale factor.
- The Big Bang was the last bounce (Big Bounce).

β/β_{cr}	Number of bounces
0.996	1
0.984	2
0.965	3
0.914	5
0.757	10



S. Desai & NP, PLB 755, 183 (2016)

It is possible to find a scalar field potential which generates the same time dependence of the scale factor.



$\beta/\beta_{cr} = 0.9998$

Plateau-like (slow-roll) potential – favored by Planck satellite data.

Torsion reproduces the same shape without hypothetical scalar fields and with only 1 parameter: particle production rate.

It predicts the CMB parameters consistent with the data.

Every black hole creates a new universe? Our Universe originated in a black hole?

Every black hole may create a new, closed, baby universe (Novikov, Pathria, Hawking, Smolin, NP).

Accordingly, our Universe may be closed and may have born in the interior of a black hole existing in a parent universe.
NP, PLB 694, 181 (2010)

This hypothesis should solve the black hole information paradox: the information goes through the Einstein-Rosen bridge to the baby universe on the other side of the black hole's event horizon.

The motion through an event horizon is one way only: it defines the past and future. Time asymmetry at the event horizon may induce time asymmetry everywhere in the baby universe and explain why time flows in one direction.

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Resumen: el Universo en un agujero negro

- La ley de conservación del momento angular total en el espacio-tiempo curvo, consistente con la ecuación de Dirac, requiere torsión. La teoría más simple con torsión, la gravedad de Einstein-Cartan, tiene el mismo Lagrangiano que RG, pero la conexión afín contiene el tensor de torsión, generado por el espín.

- El colapso gravitacional de una esfera esféricamente simétrica de un fluido de espín crea un horizonte de eventos. La materia dentro del horizonte se colapsa a densidades extremadamente altas, en las que la torsión actúa como repulsión gravitacional.

- Sin cizallamiento, la torsión evita una singularidad y la reemplaza por un rebote no singular. Con el cizallamiento, la torsión evita una singularidad si el número de fermiones aumenta durante la contracción a través de la producción de partículas cuánticas.

- La producción de partículas durante la expansión produce enormes cantidades de materia y puede generar un período finito de inflación. El universo cerrado resultante en el otro lado del horizonte de eventos puede tener varios rebotes. Tal universo es oscilatorio, con cada ciclo de mayor tamaño que el ciclo anterior, hasta que alcanza el tamaño cosmológico y se expande indefinidamente.