Quantum information with photons: fractional topological phase of qudits

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XIX Encuentro de Física– 26/09/2020

Students: 44000;
Professors: 3200

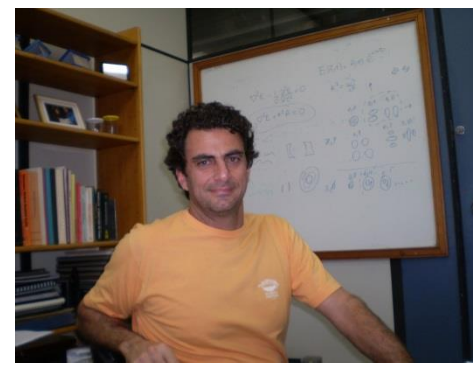
A. A. Matoso[1], R. A. Ribeiro, X. Sánchez-Lozano[1], W. M. Pimenta[1], P. Machado[1], B. Marques[1], L. E. Oxman[2], A. Z. Khoury[2], F. Sciarrino[3] , S. Pádua[1]





1 Universidade Federal de Minas Gerais, Belo Horizonte, Brazil.
2 Universidade Federal Fluminense, Niteroi, Brazil.
3 Università di Roma, La Sapienza, Roma, Italy.





Summary

- Two photon source
- Qudits
- Fractional topological phase (FTF) for two qudits from the polar decomposition
- How to measure FTF through the coincidence counts of a entangled photon pair.
- Using Hyperentangled states.
- Noise added and robustness of the fractional topological phase.
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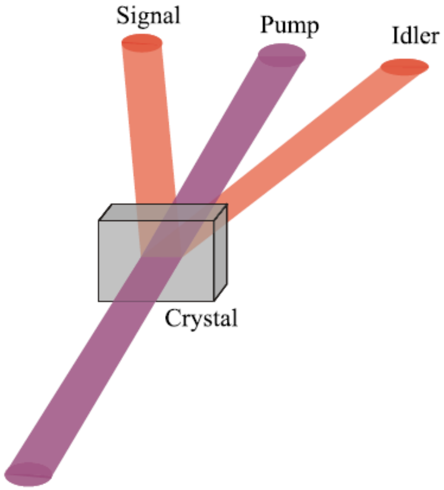
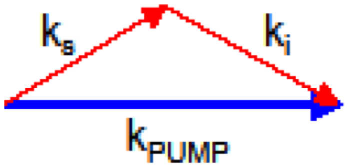
Spontaneous parametric Down-Conversion

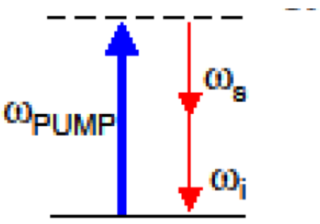
Spontaneous parametric down conversion





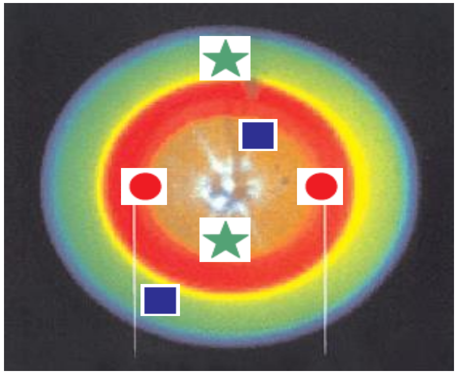
$$\hbar\vec{k}_p = \hbar\vec{k}_s + \hbar\vec{k}_i$$

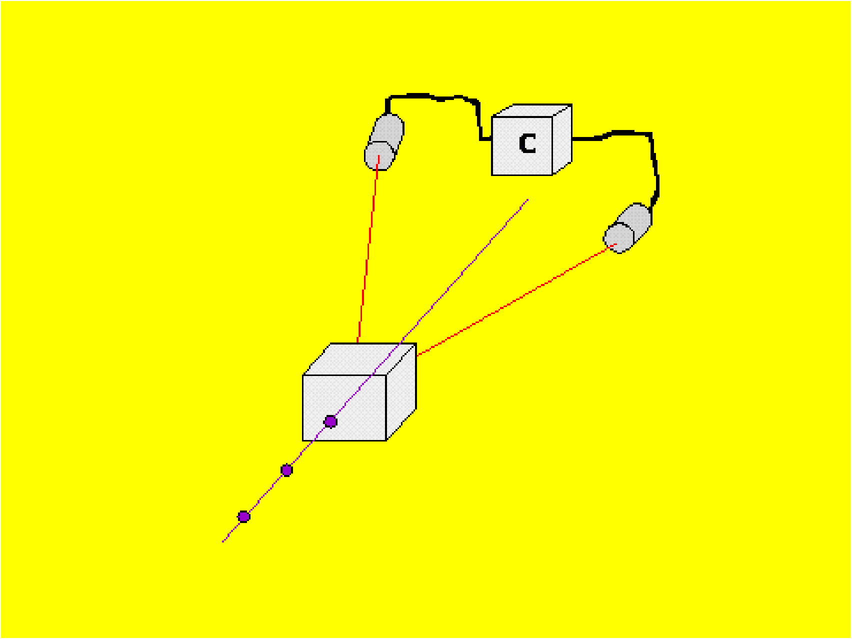
$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$



$\varphi_{PUMP} = \varphi_s + \varphi_i$

Twin Photons generator





P = pump
S = signal
I = Idler

Paulo H. Souto Ribeiro - UFRJ

http://en.wikipedia.org/wiki/Spontaneous_parametric_down-conversion

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Qudits: Quantum system with d levels.

^{87}Rb

$d{=}8$ qudit

Dianne P. O'Leary et. al. , PRA **74**, 032334 (2006).

Fótons: Orbital angular momentum, polarization, time, linear momentum.

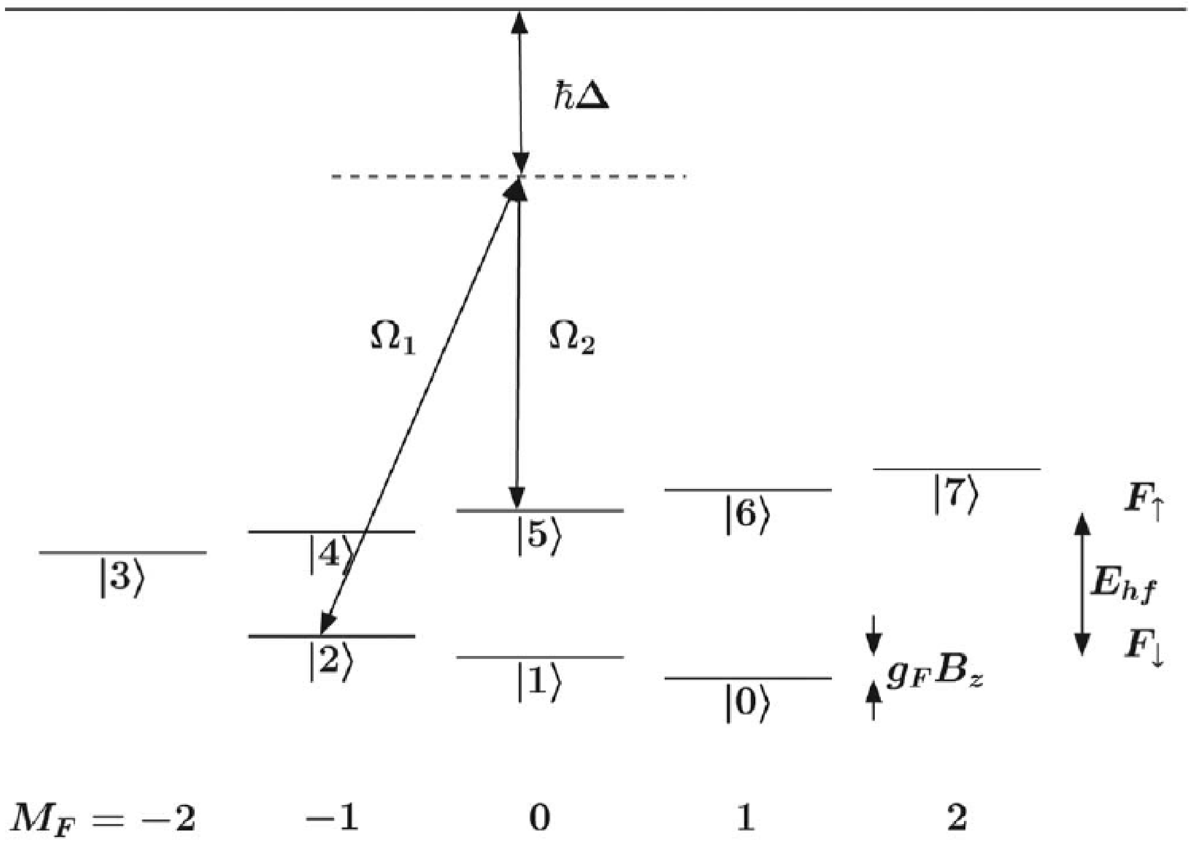


FIG. 1. A single $d{=}8$ qudit encoded in the ground-state hyperfine levels of ^{87}Rb. A pair of lasers can couple states in different hyperfine manifolds according to the selection rule $\Delta M_F{=}0, \pm 1$.

On-chip generation of high-dimensional entangled quantum states and their coherent control

Michael Kues,[1,2,+,*] Christian Reimer,[1,*] Piotr Roztocki,[1] Luis Romero Cortés,[1] Stefania Sciara,[1,3] Benjamin Wetzel,[1,4] Yanbing Zhang,[1] Alfonso Cino,[3] Sai T. Chu,[5] Brent E. Little,[6] David J. Moss,[7] Lucia Caspani,[8,9] José Azaña,[1] and Roberto Morandotti[1,10,11+]

[1]Institut National de la Recherche Scientifique - Centre Énergie, Matériaux et Télécommunications (INRS-EMT),

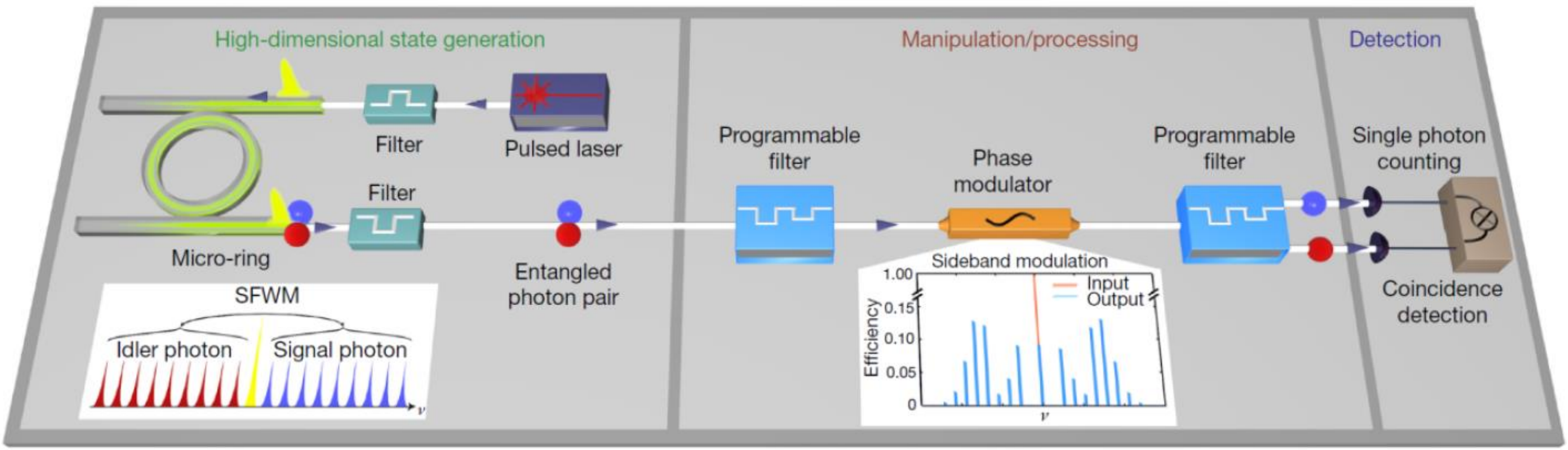


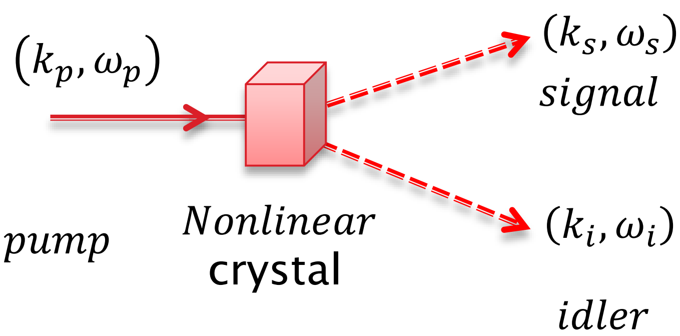
Figure 1 | Experimental setup for high-dimensional quantum state generation and control.

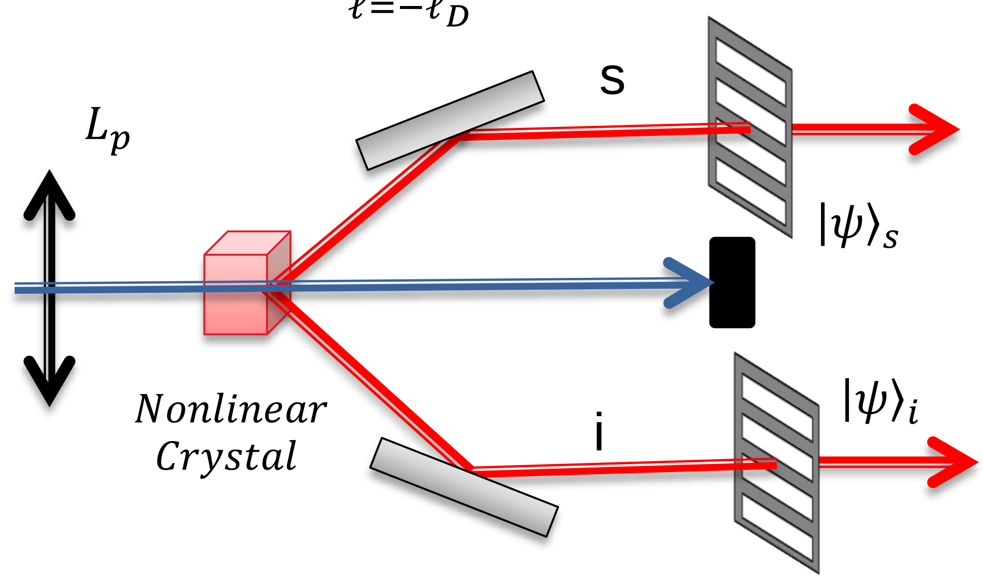
Nature **volume546**, pages622–626 (29 June 2017)

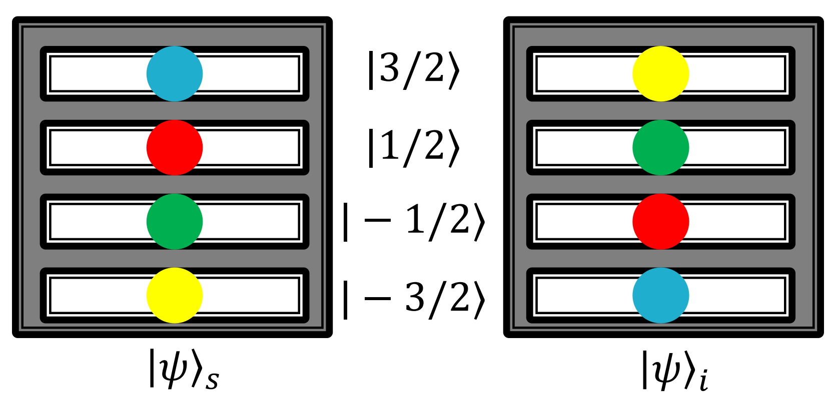
Qudit: d-level quantum system.

Two-photons entangled in transversal path: 2 qudits

- $|\psi\rangle = \frac{1}{\sqrt{D}} \sum_{\ell=-\ell_D}^{\ell_D=(D-1)/2} |\ell\rangle_s \otimes |-\ell\rangle_i$







L. Neves, et. al. PRA, **69**, 042305 (2004)
L. Neves, et. al. PRL, **94**,100501 (2005)

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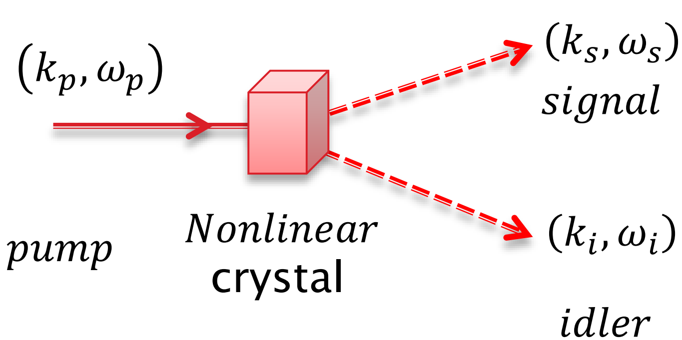
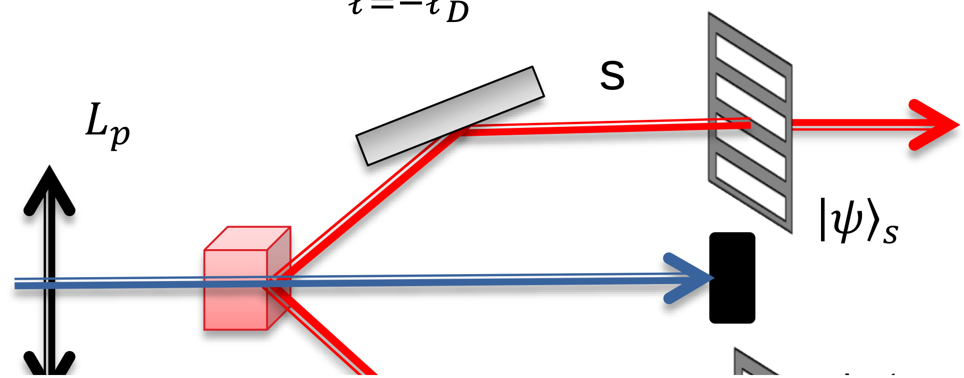


- $|\psi\rangle = \frac{1}{\sqrt{D}} \sum_{\ell=-\ell_D}^{\ell_D=(D-1)/2} |\ell\rangle_s \otimes |-\ell\rangle_i$



$$|\Psi\rangle_4 = \frac{1}{2}\left[\left|+\tfrac{1}{2}\right\rangle_s \left|-\tfrac{1}{2}\right\rangle_i + \left|-\tfrac{1}{2}\right\rangle_s \left|+\tfrac{1}{2}\right\rangle_i + e^{i\frac{kd^2}{z_A}} \left(\left|-\tfrac{3}{2}\right\rangle_s \left|+\tfrac{3}{2}\right\rangle_i + \left|+\tfrac{3}{2}\right\rangle_s \left|-\tfrac{3}{2}\right\rangle_i \right) \right]$$

L. Neves, et. al. PRA, **69**, 042305 (2004)
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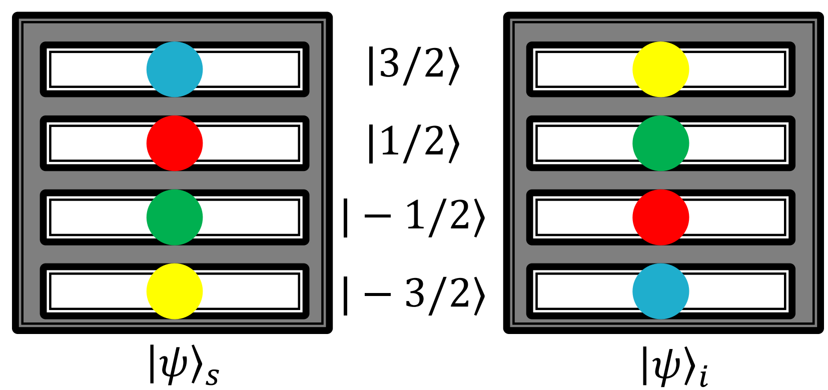

Generation of Entangled States of Qudits using Twin Photons

Leonardo Neves,[1] G. Lima,[1] J. G. Aguirre Gómez,[1,2] C. H. Monken,[1] C. Saavedra,[2] and S. Pádua[1,*]
[1]*Departamento de Física, Universidade Federal de Minas Gerais, Caixa Postal 702, Belo Horizonte, Minas Gerais 30123-970, Brazil*
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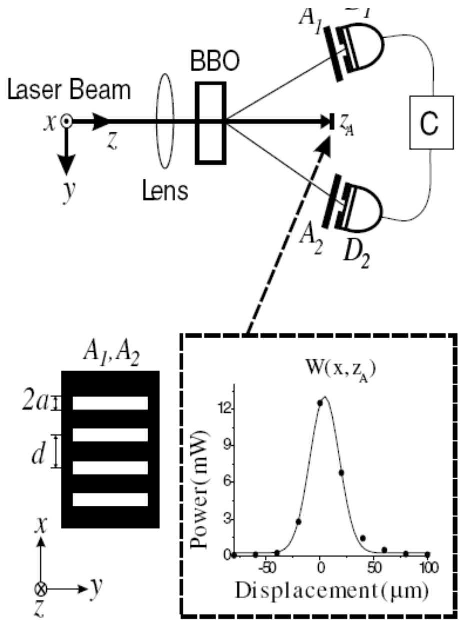

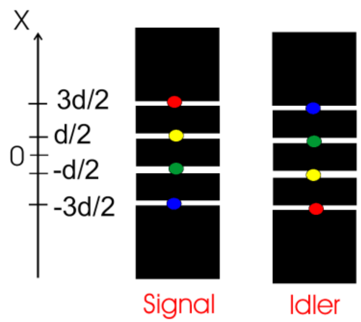

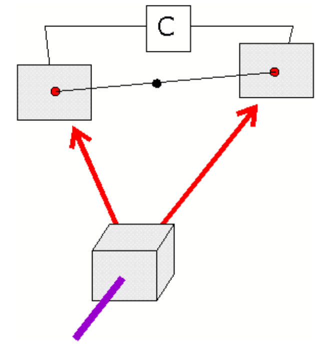

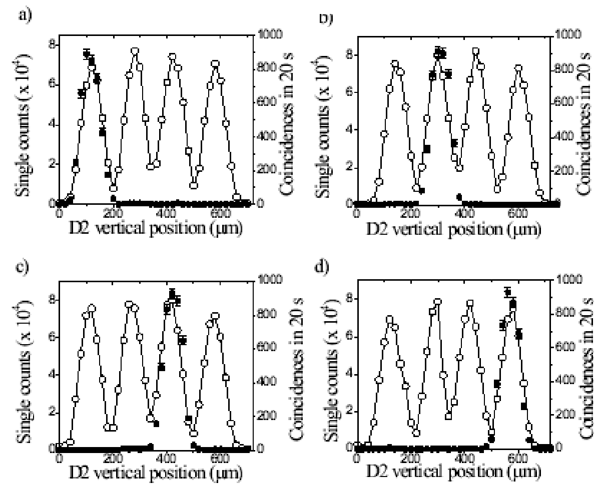

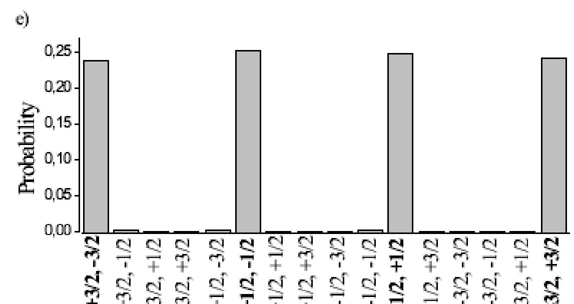

$$|\Psi\rangle = 0,50|-\tfrac{1}{2}, +\tfrac{1}{2}\rangle + 0,50|+\tfrac{1}{2}, -\tfrac{1}{2}\rangle + e^{i(kd^2/z_A)}(0,49|-\tfrac{3}{2}, +\tfrac{3}{2}\rangle + 0,49|+\tfrac{3}{2}, -\tfrac{3}{2}\rangle), \qquad (5)$$

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PHYSICAL REVIEW A **87**, 042113 (2013)

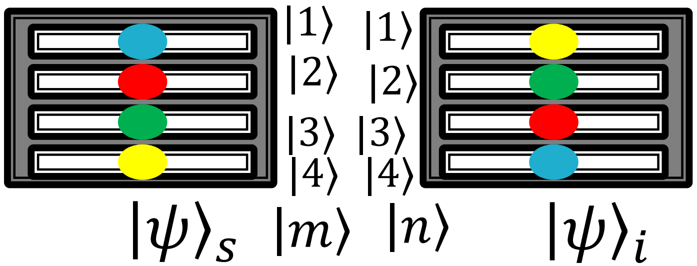
Fractional topological phase on spatially encoded photonic qudits

A. Z. Khoury,[1] L. E. Oxman,[1] B. Marques,[2] A. Matoso,[2] and S. Pádua[2]
[1] *Instituto de Física, Universidade Federal Fluminense, 24210-346 Niterói, Rio de Janeiro, Brazil*
Departamento de Física, Universidade Federal de Minas Gerais, 31270-901 Belo Horizonte, Minas Gerais, Brazil
(Received 17 January 2013; published 19 April 2013)

L. E. Oxman and A. Z. Khoury, Physical Review Letters, 106:240503, 2011.

E. Sjöqvist, Int. J. Quantum Chem. **115**, 1311 (2015)

Topological Phase



Example: A two-qutrit (d = 3) state ;

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|1,3\rangle + |2,2\rangle + |3,1\rangle) \qquad \alpha = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Let´s consider the most general two-qudit pure state:

$$|\psi\rangle = \sum_{m,n=1}^{d} \alpha_{mn}\,|m,n\rangle$$

and organize the coefficients on a matrix α, which admits the polar decomposition:

$$|\psi\rangle(0) \Longrightarrow \quad \alpha(0) = e^{i\phi(0)} S(0) Q(0)$$

Q ⇨ Hermitian matrix $Q = \overline{Q^T}$

S ⇨ SU(d) matrix $S^{-1} = \overline{S^T}$
det(S) = 1

d ⇨ One-qudit dimension

A. Z. Khoury, L. E. Oxman, B. Marques, A. Matoso, and S. Pádua; Phys. Rev. A, 87:042113, 2013.

Topological Phase

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Q ➡ Hermitian matrix

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d ➡ One qudit dimension

Now we consider an evolution given by local unitary operations, which preserves the polar decomposition

$$\boxed{\alpha(t) = U_s(t)\alpha(0)V_i^{\mathsf{T}}(t)}$$

$$\boxed{\alpha(t) = e^{i\phi(t)} Q(t) S(t)}$$

- and finally we consider a cyclic evolution

$$\boxed{\alpha(T) = e^{i\gamma}\alpha(0).}$$

[2] A. Z. Khoury, L. E. Oxman, B. Marques, A. Matoso, and S. Padua; Phys. Rev. A, 87:042113, 2013.

$$\alpha(T) = e^{i\gamma}\alpha(0)$$

+

$$\gamma = \Delta\phi + \gamma_q + \gamma_s$$

Topological Phase

Using the polar decomposition and spliting γ for each sector of it:

$$e^{i\phi(T)}S(T)Q(T) = \left(e^{i\Delta\phi}e^{i\phi(0)}\right)\left(e^{i\gamma_s}S(0)\right)\left(e^{i\gamma_q}Q(0)\right)$$

From the properties of the Hermitian and the SU(d) matrices, one can show that:

$$\gamma_q = 0 \quad \gamma_s = \frac{2l\pi}{d} \qquad l = 0, 1, 2, ..., d-1$$

Fractional topological Phase $\hookrightarrow$ FTP

So, the phase acquired after a cycle is:

$$\gamma = \arg\{\mathrm{Tr}[\alpha^\dagger(0)\alpha(T)]\} = \Delta\phi + \frac{2l\pi}{d}$$

$$\langle\psi \mid \psi\rangle = \mathrm{Tr}[\alpha^\dagger\alpha] = 1$$

[3] Pérola Milman and Rémy Mosseri. Physical Review Letters, 90:230403, 2003.

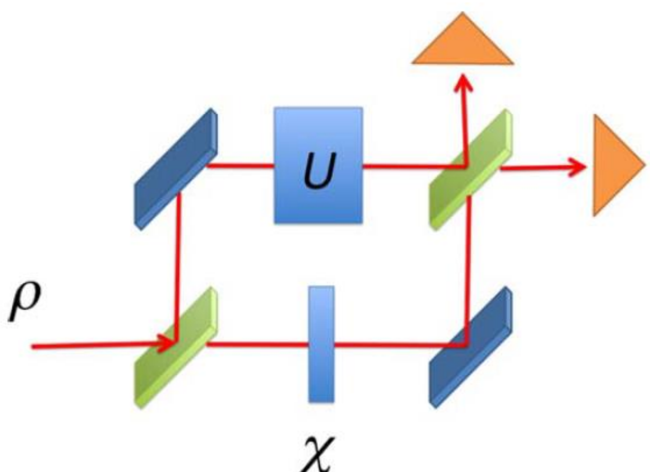
How to measure the FTP?

Interference between a non-evolved and an evolved state:

$$C = \left|\left|\frac{\alpha(t)+\alpha(0)}{\sqrt{2}}\right|\right|^2 = 1 + \mathrm{Re}\{\mathrm{Tr}[\alpha^\dagger(0)\underline{\alpha(t)}]\} = 1 + v\cos(\Delta\phi + \gamma')$$

$$v = |\mathrm{Tr}[\alpha^\dagger(0)\alpha(t)]|,$$

$$\boxed{\Delta\phi + \gamma' = \arg\{\mathrm{Tr}[\alpha^\dagger(0)\alpha(t)]\}}$$



E. Sjöqvist, Int. J. Quantum Chem. **115**, 1311 (2015)

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$$v = |\mathrm{Tr}[\alpha^\dagger(0)\alpha(t)]|, \qquad \Delta\phi + \gamma' = \arg\{\mathrm{Tr}[\alpha^\dagger(0)\alpha(t)]\}$$

$$\boxed{\alpha(t) = U_s(t)\alpha(0)V_i^{\mathsf{T}}(t)} \qquad t \in [0,1]$$

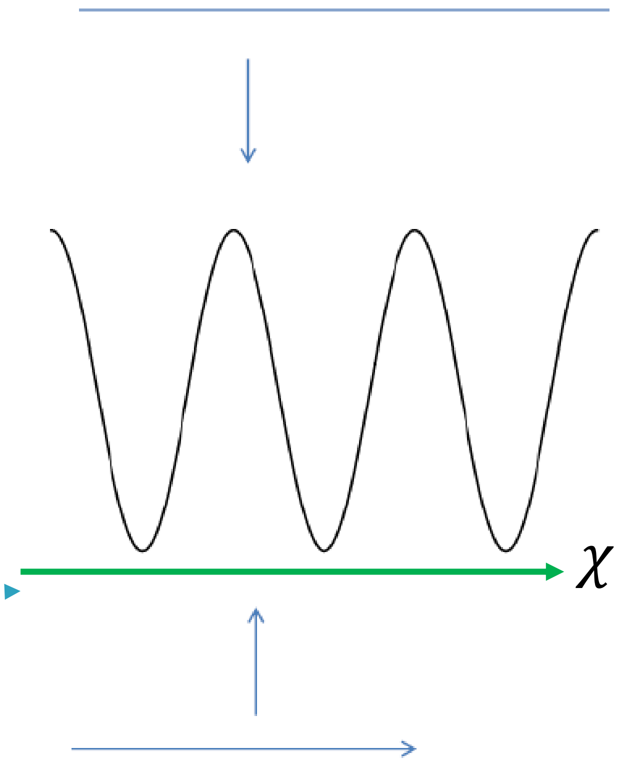
Let's take
U_s and V_i diagonals

$$\bar{U}(0) = \bar{V}(0) = \mathbb{1}$$

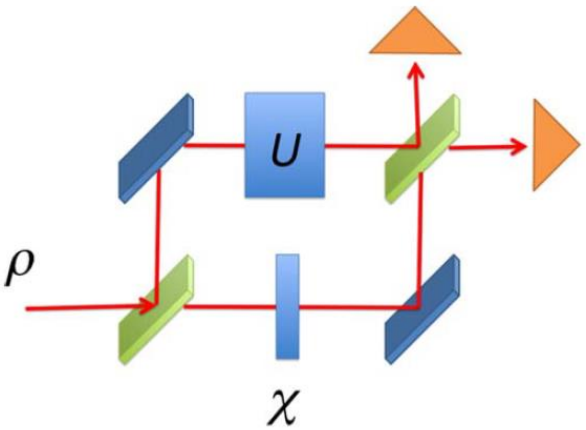
$$\overline{U}_s(t) = \begin{pmatrix} e^{i\xi_1(t)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{i\xi_m(t)} \end{pmatrix} \qquad \overline{V}_i(t) = \begin{pmatrix} e^{i\chi_1(t)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{i\chi_m(t)} \end{pmatrix}$$

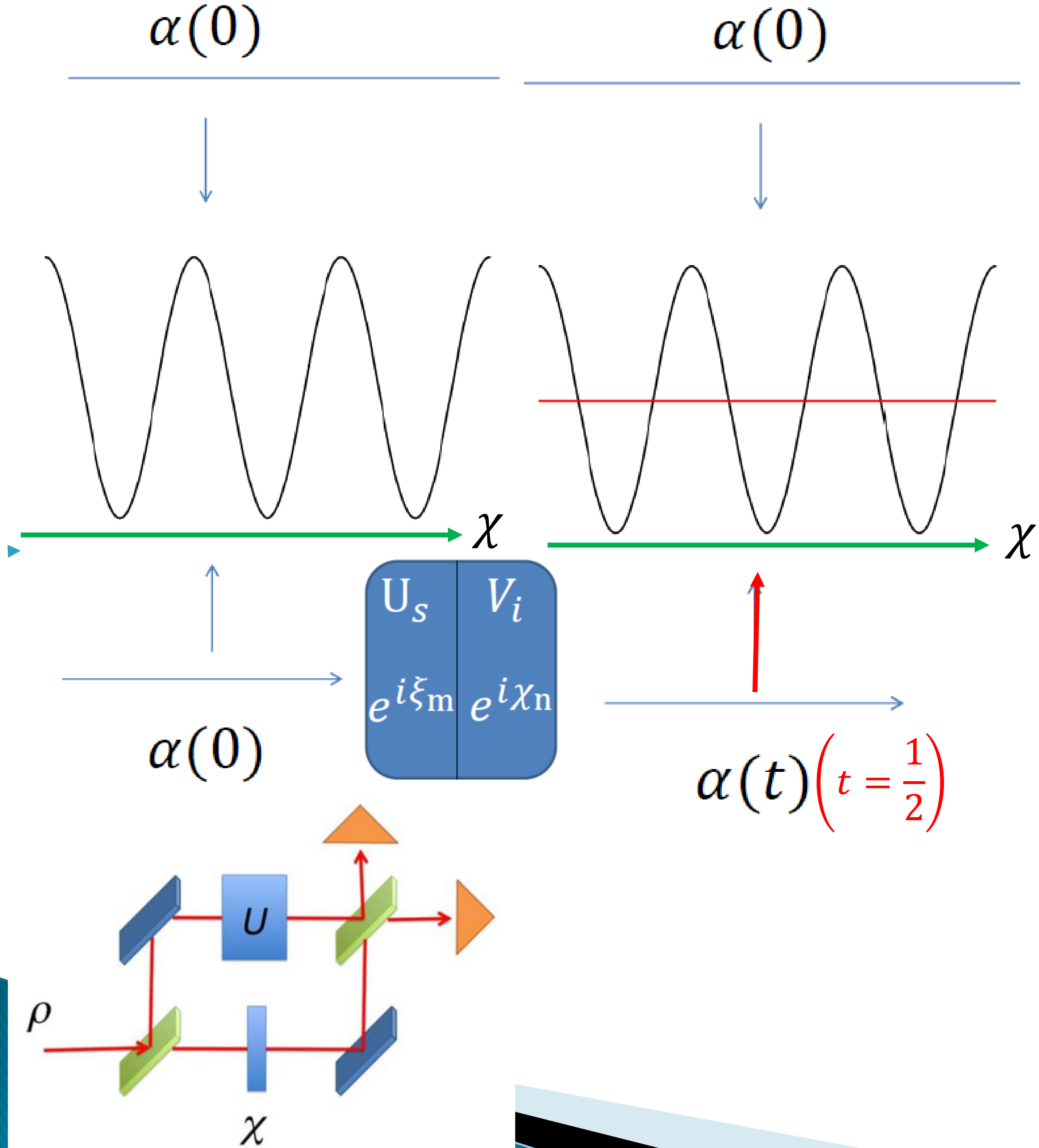
SU(d) operations $\quad \sum_{m=1}^{d} \xi_m = \sum_{n=1}^{d} \chi_n = 0$

$\alpha(0)$

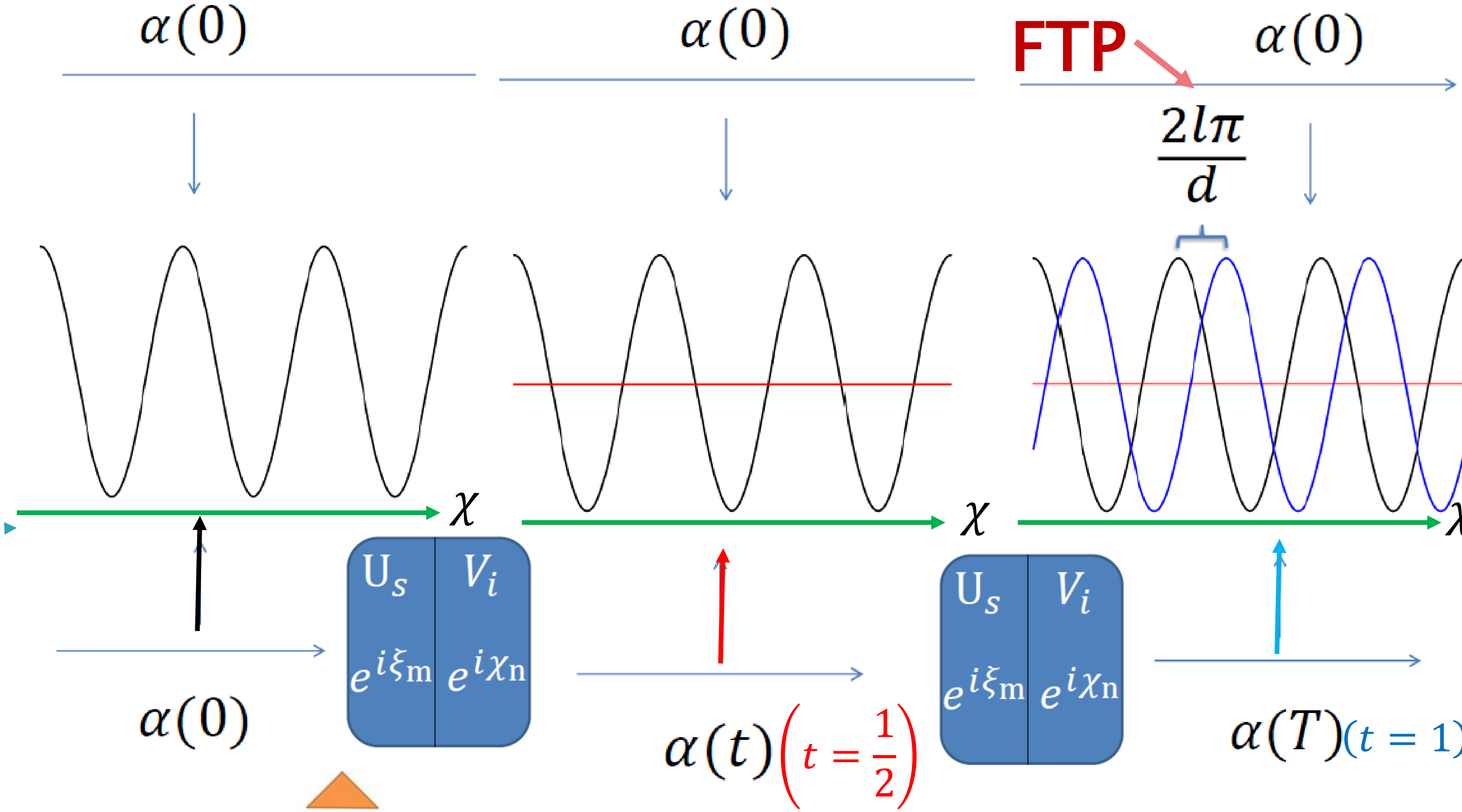


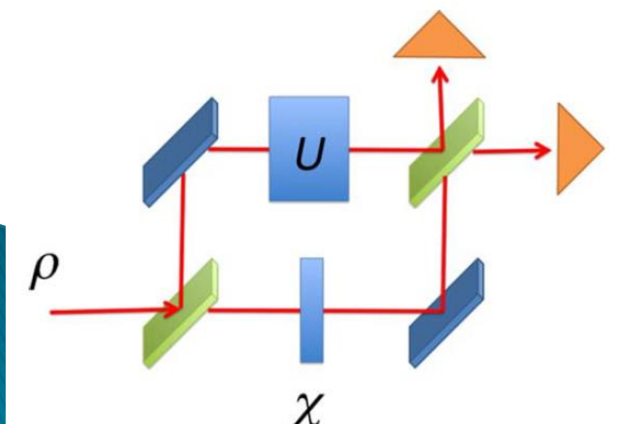
$\alpha(0)$





$\alpha(0)$
$\alpha(0)$
χ
χ
U_s
V_i
$e^{i\xi_m}$
$e^{i\chi_n}$
$\alpha(0)$
$\alpha(t)\left(t = \frac{1}{2}\right)$
U
ρ
χ





$$v = |\mathrm{Tr}[\alpha^\dagger(0)\alpha(t)]|,$$

PHYSICAL REVIEW A **94**, 052305 (2016)

Experimental observation of fractional topological phases with photonic qudits

A. A. Matoso,[1] X. Sánchez-Lozano,[1,2] W. M. Pimenta,[1,3] P. Machado,[1] B. Marques,[1,4] F. Sciarrino,[5] L. E. Oxman,[6] A. Z. Khoury,[6] and S. Pádua[1]

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[2]*Departamento de Física - DCI, Universidad de Guanajuato P. O. Box E-143, 37150, León, Gto., México*

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[4]*Instituto de Física, Universidade de São Paulo, 05508-090 São Paulo, Brazil*

[5]*Dipartimento di Física, Università di Roma La Sapienza, 00185 Roma, Italy*

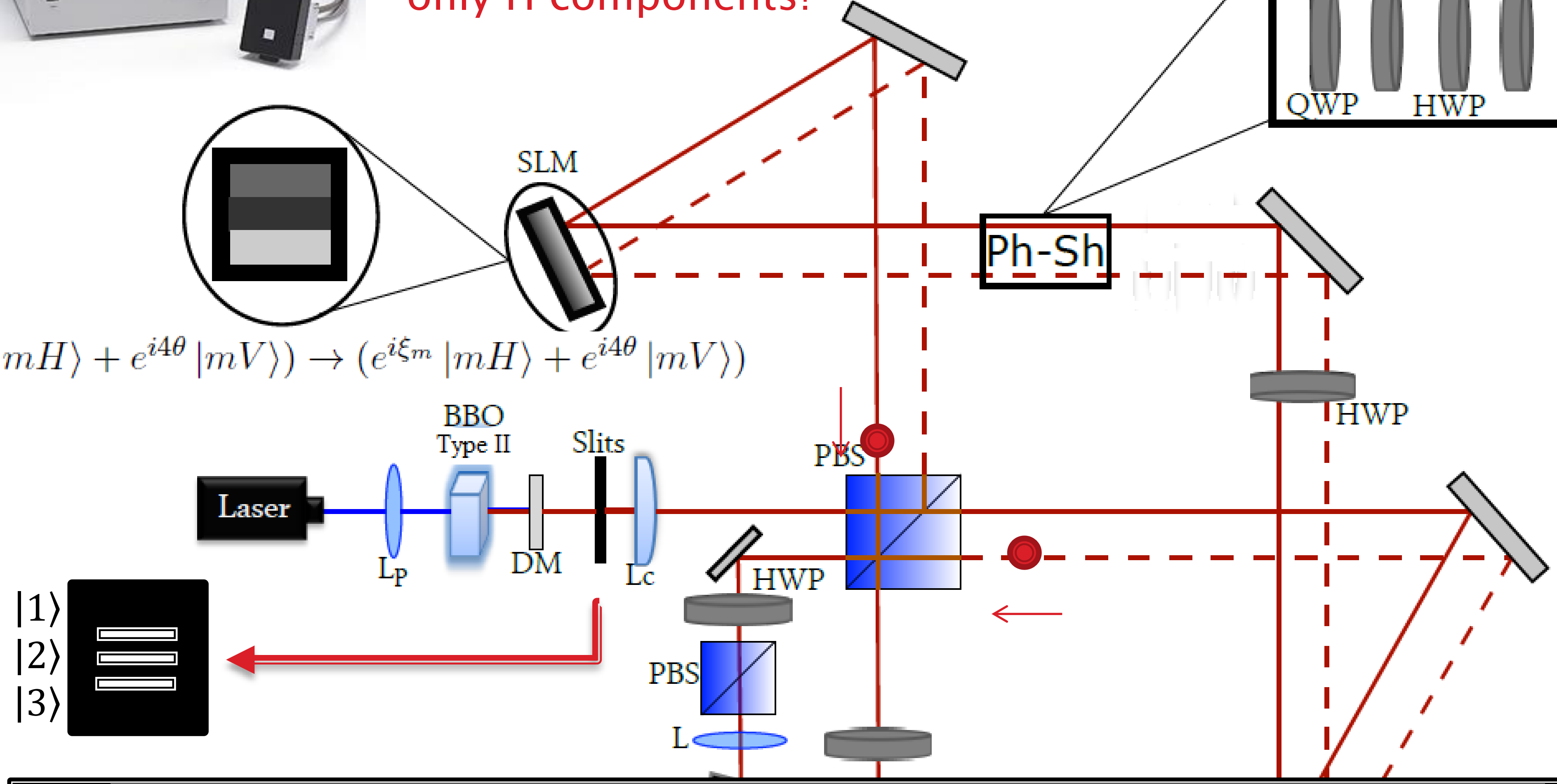
[6]*Instituto de Física, Universidade Federal Fluminense, 24210-346 Niterói, Rio de Janeiro, Brazil*

(Received 30 November 2015; published 4 November 2016)

Geometrical and topological phases play a fundamental role in quantum theory. Geometric phases have been proposed as a tool for implementing unitary gates for quantum computation. A fractional topological phase has been recently discovered for bipartite systems. The dimension of the Hilbert space determines the topological phase of entangled qudits under local unitary operations. Here we investigate fractional topological phases acquired by photonic entangled qudits. Photon pairs prepared as spatial qudits are operated inside a Sagnac interferometer and the two-photon interference pattern reveals the topological phase as fringes shifts when local operations are performed. Dimensions $d = 2$, 3, and 4 were tested, showing the expected theoretical values.

how to measure the FTF

This SLMs modulates only H components!



$$(|mH\rangle + e^{i4\theta}|mV\rangle) \rightarrow (e^{i\xi_m}|mH\rangle + e^{i4\theta}|mV\rangle)$$

$$|\psi_1\rangle = \frac{-i}{2}\sum_{m,n}\alpha_{mn}(e^{i\xi_m}|mH,nH\rangle - e^{i\xi_m}|mH,nV\rangle + e^{i4\theta}|mV,nH\rangle - e^{i4\theta}|mV,nV\rangle).$$

$\alpha(t)$ D2 $\alpha(0)$

how to measure the FTF

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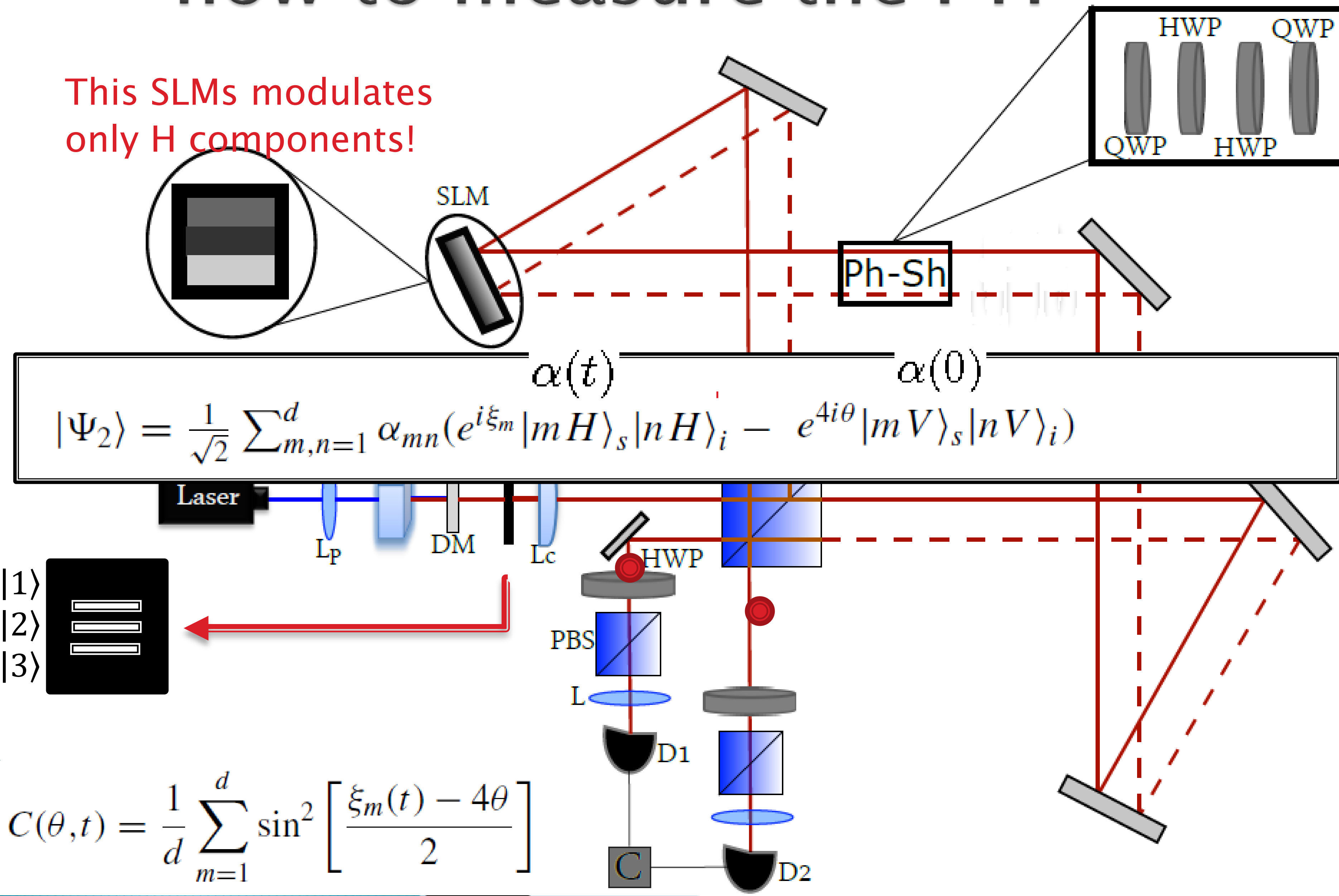
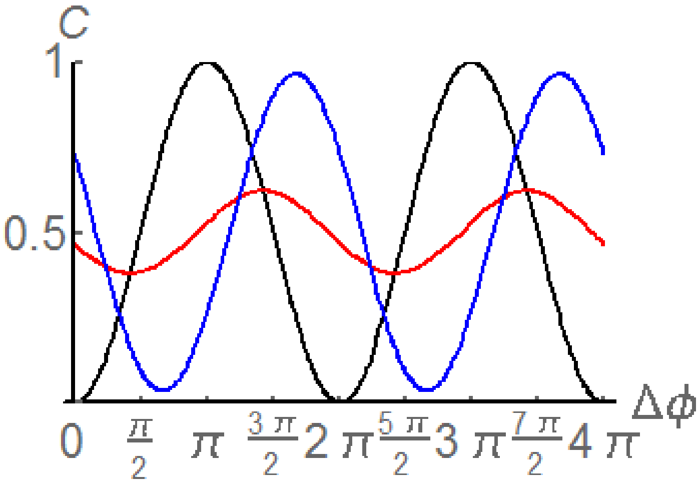


$$|\Psi_2\rangle = \frac{1}{\sqrt{2}} \sum_{m,n=1}^{d} \alpha_{mn} \left(e^{i\xi_m} |mH\rangle_s |nH\rangle_i - e^{4i\theta} |mV\rangle_s |nV\rangle_i\right)$$

$$C(\theta,t) = \frac{1}{d} \sum_{m=1}^{d} \sin^2\left[\frac{\xi_m(t) - 4\theta}{2}\right]$$

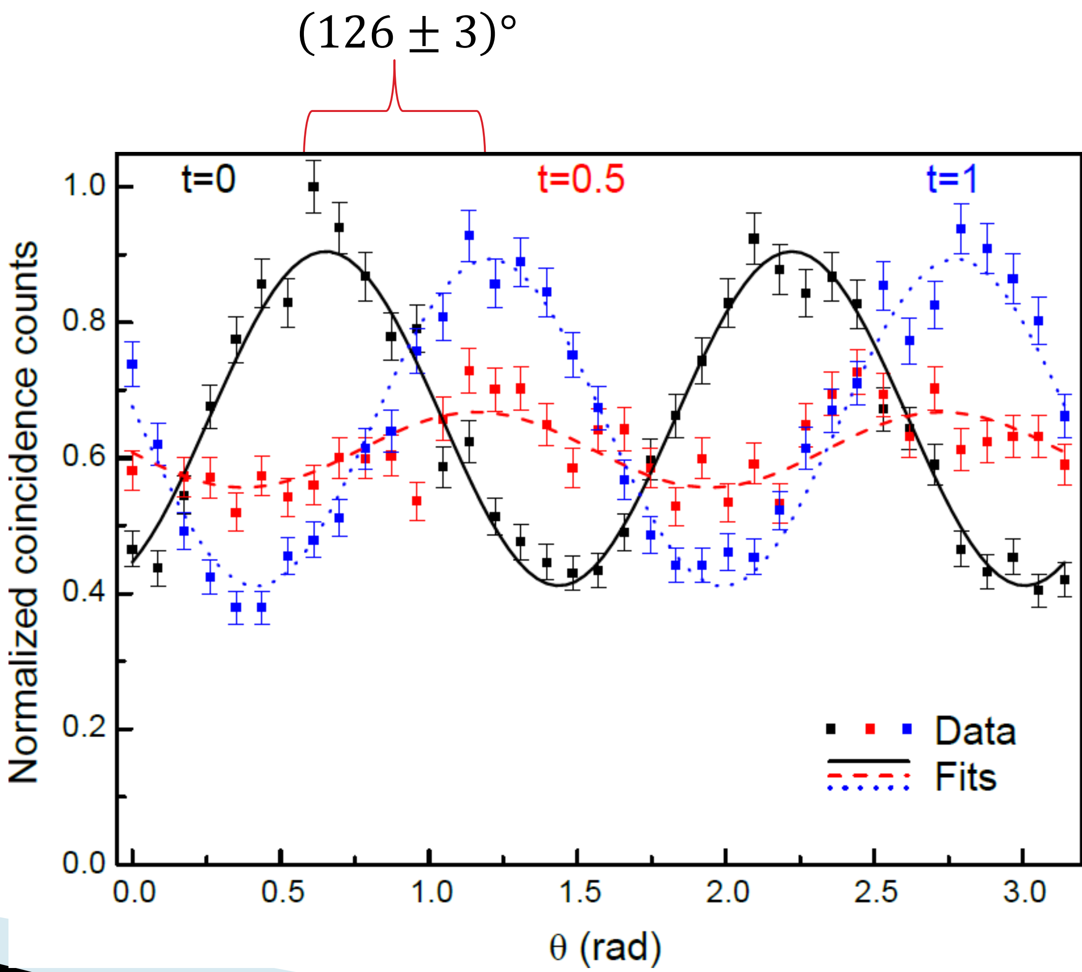
Experimental Results



Theoretical Curve recalculated for the measured population.

$$\gamma_s = \frac{2/\pi}{d}$$

Qutrit



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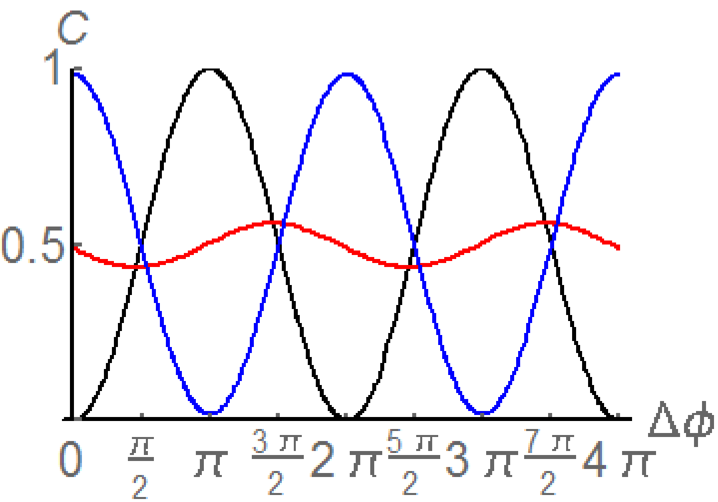
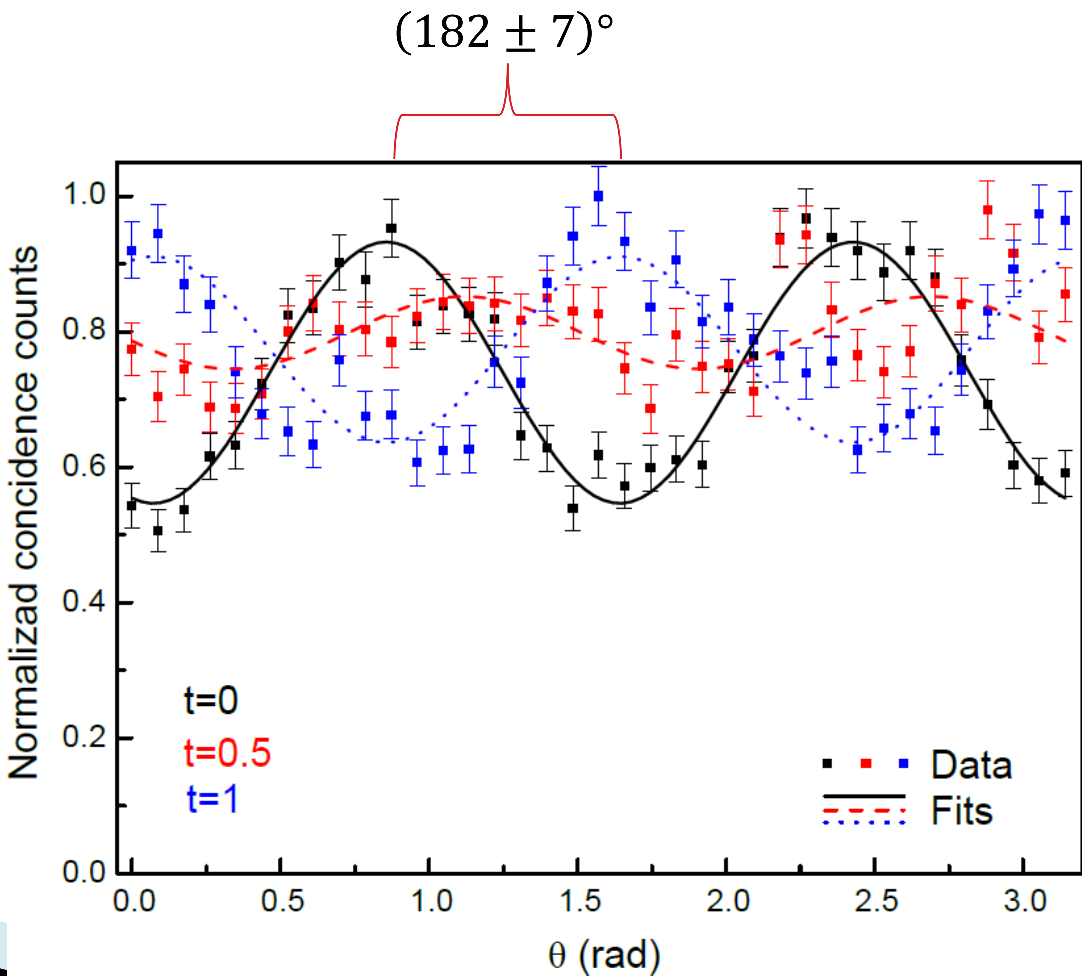


Qubit

$$\gamma_s = \frac{2l\pi}{d}$$



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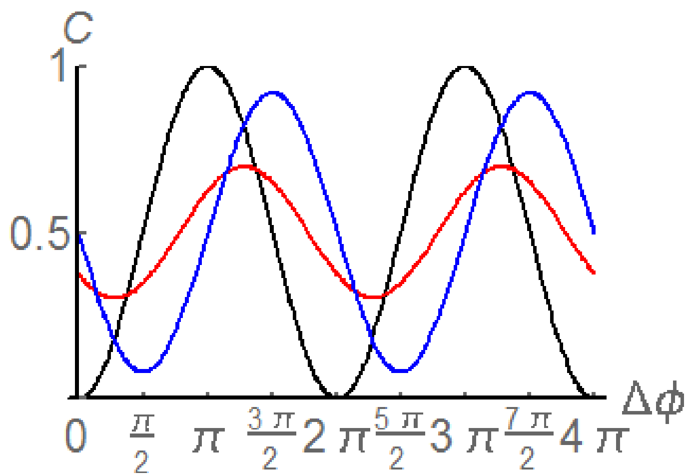
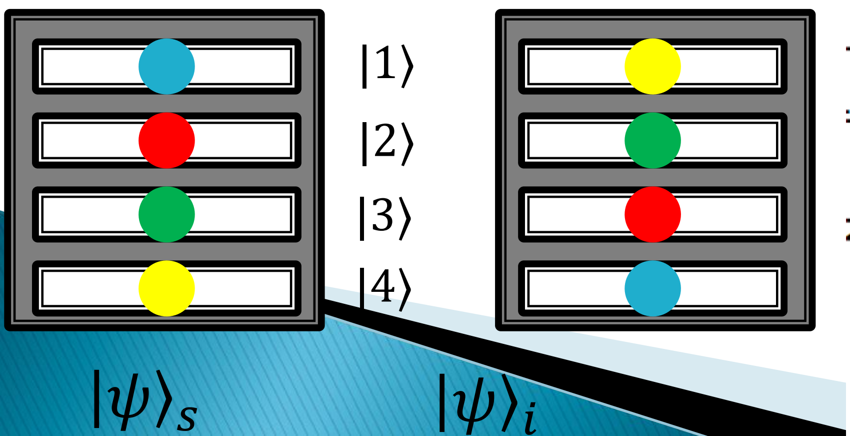
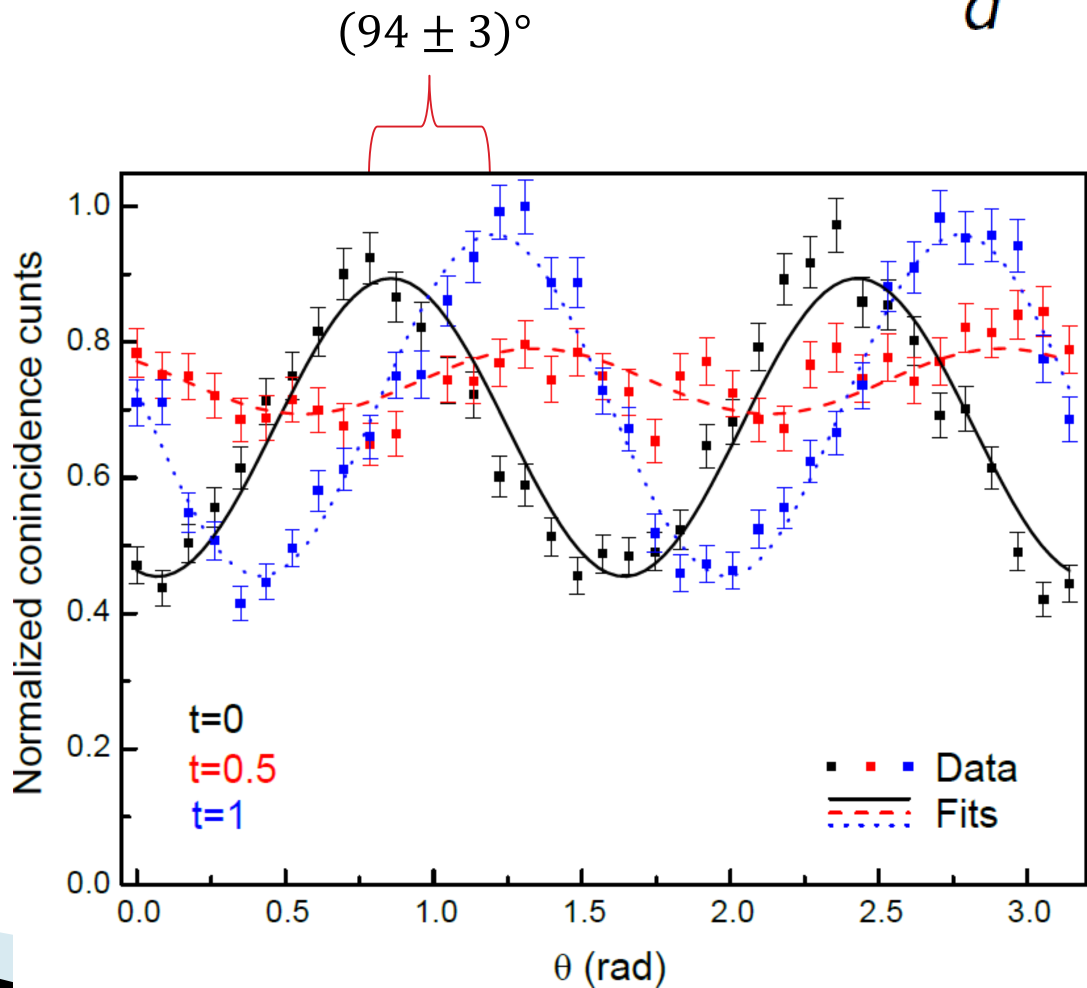
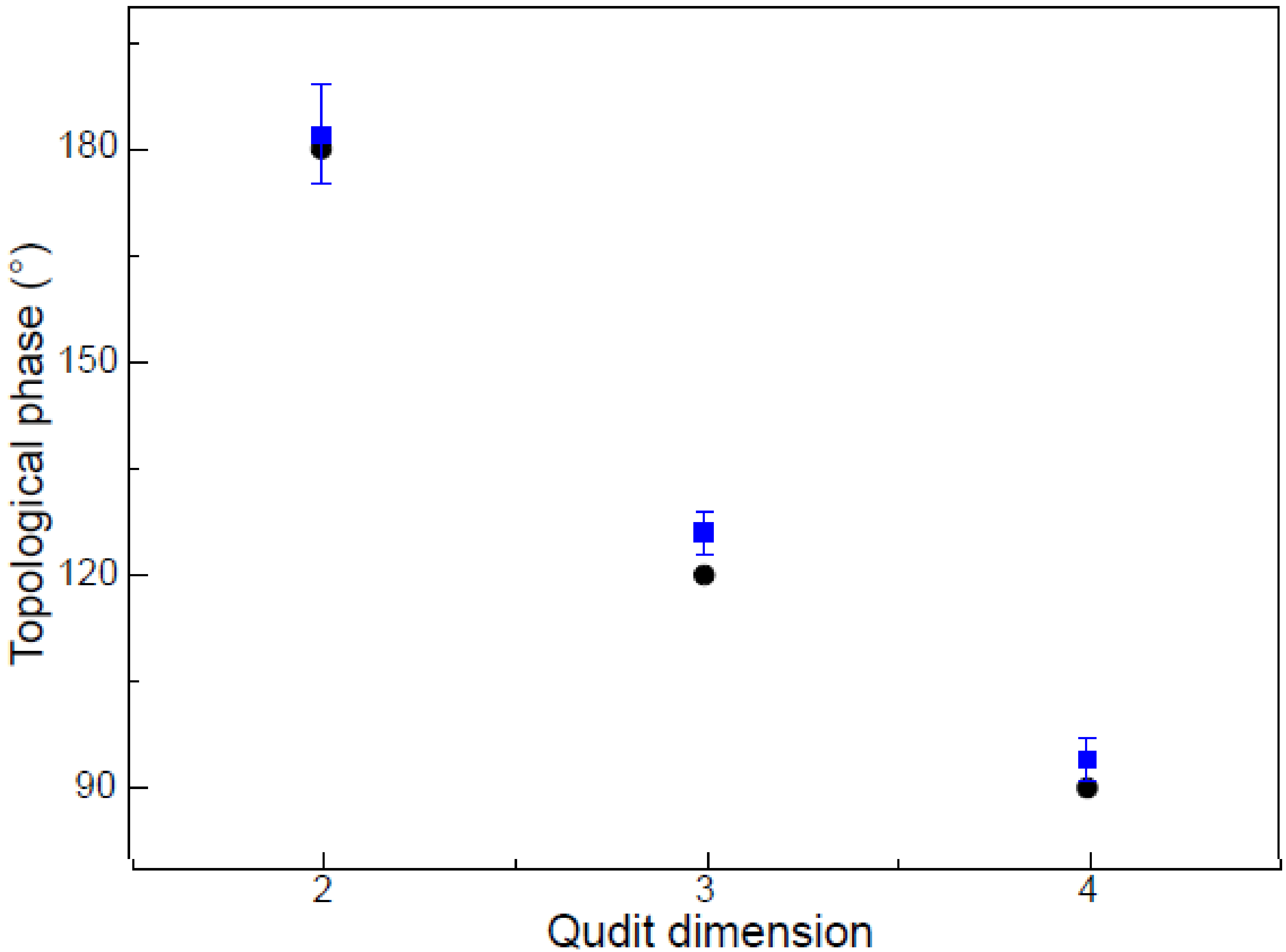


$$\gamma_s = \frac{2l\pi}{d}$$

Ququart







A. A. Matoso, X. Sánchez-Lozano, W. M. Pimenta, P. Machado, B. Marques, F. Sciarrino, L. E. Oxman, A. Z. Khoury, and S. Pádua, Phys. Rev. A, 94:052305, Nov 2016

Summary

- Qudits
- Fractional topological phase (FTF) for two qudits from the polar decomposition
- How to measure FTF through the coincidence counts of a entangled photon pair.
- Using Hyperentangled states.
- Noise added and robustness of the fractional topological phase.
- Conclusions

Fractional topological phase measurement with a hyperentangled photon source

A. A. Matoso[1,2,*], R. A. Ribeiro[1], L. E. Oxman[3], A. Z. Khoury[3], and S. Pádua[1]

[1]Departamento de Física, Universidade Federal de Minas Gerais, 31270-901 Belo Horizonte, Minas Gerais, Brazil
[2]Institut für Angewandte Physik, Universität Bonn, Wegelerstr. 8, 53115 Bonn, Germany
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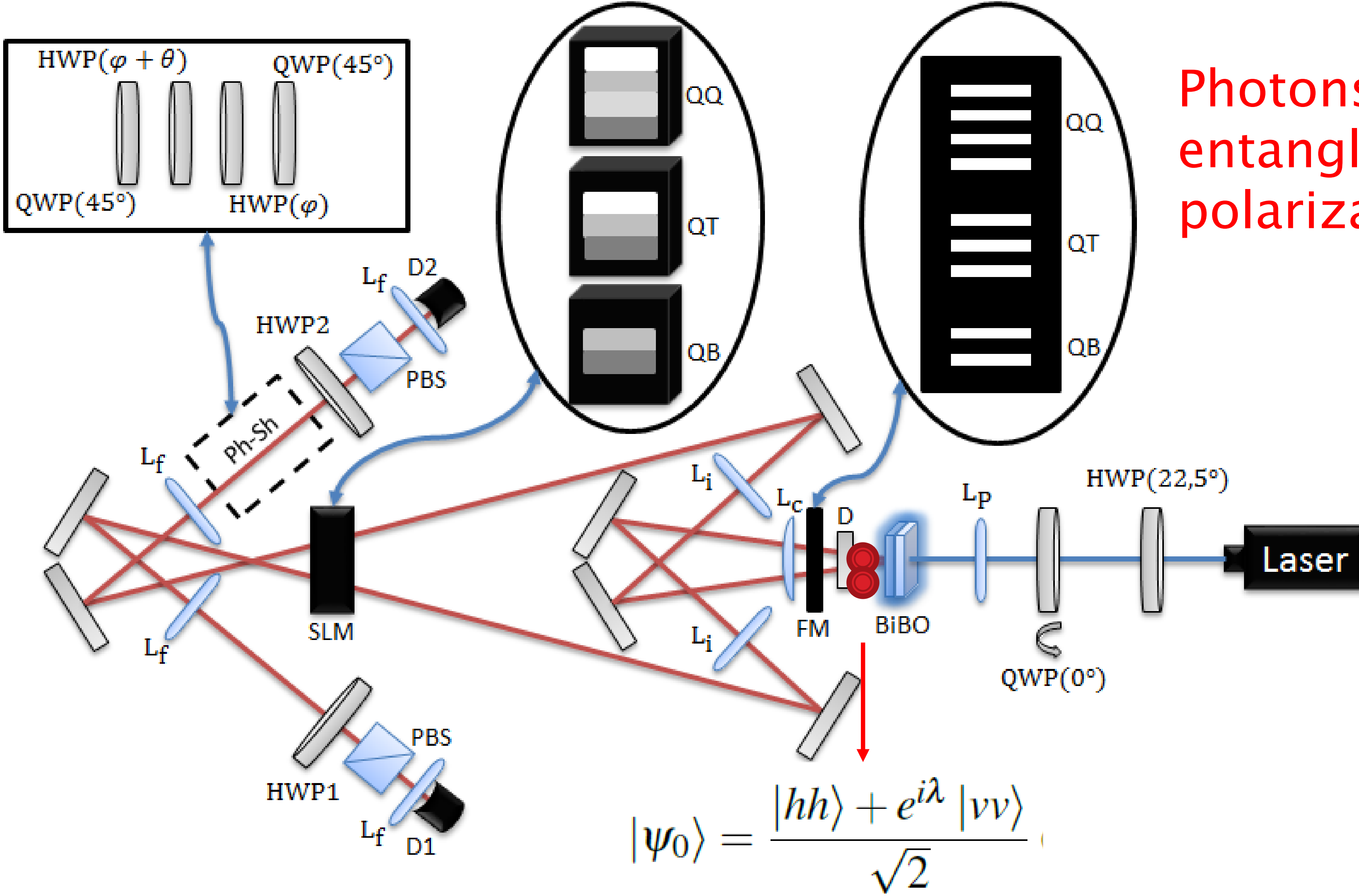
ABSTRACT

Pairs of photons simultaneously entangled in their path and polarization degrees of freedom are used to measure the topological phase acquired by bipartite entangled states. Qudits of arbitrary dimensions are encoded on the photons transverse positions while polarization entanglement is used as an auxiliary resource for quantum interference measurements. With this scheme the fractional phases predicted for dimensions $d = 2$, 3 and 4 could be measured with visibilities for the interference curves beyond the limit allowed for classical sources.

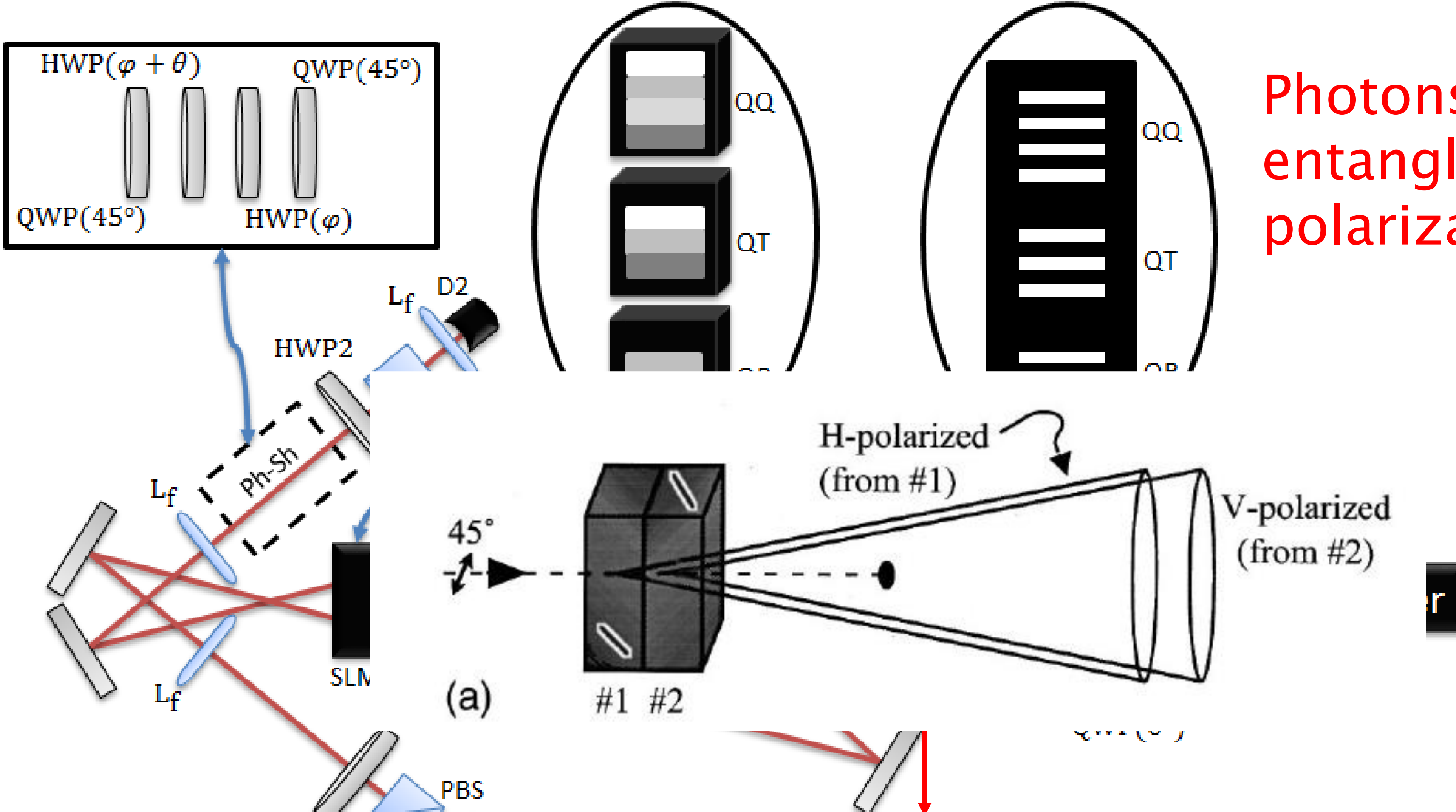
A. A. Matoso, R. A. Ribeiro, L. E. Oxman, A. Z. Khoury, and S. Pádua, Fractional topological phase measurement with a hyperentangled photon source, *Sci Rep* **9,** 577 (2019).

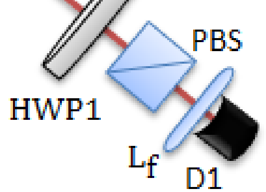


Photons entangled in polarization

$$|\psi_0\rangle = \frac{|hh\rangle + e^{i\lambda}\,|vv\rangle}{\sqrt{2}}$$

Paul G. Kwiat, Edo Waks, Andrew G. White, Ian Appelbaum, and Philippe H. Eberhard, Phys. Rev. A, 60:R773–R776, Aug 1999





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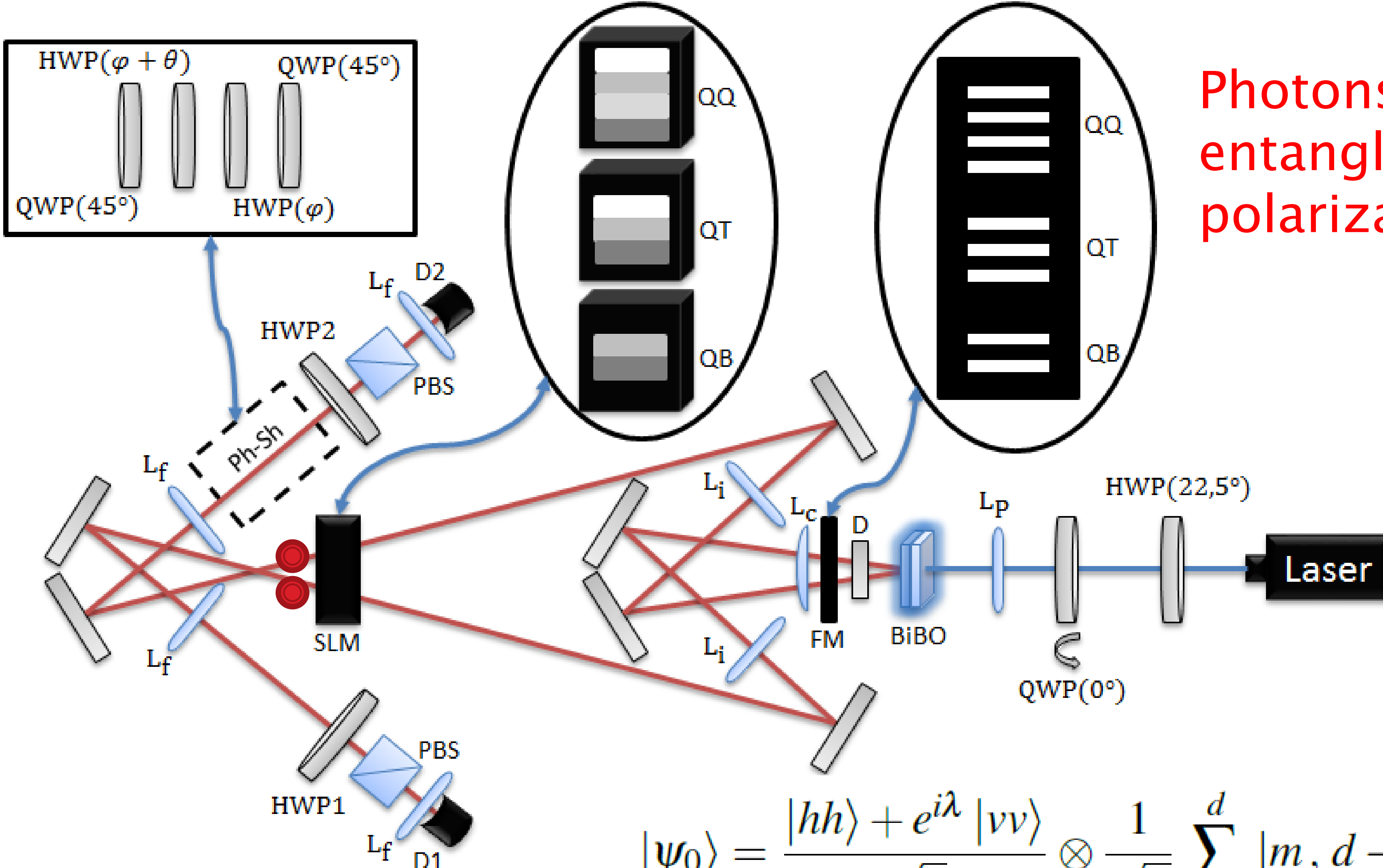


Photons entangled in polarization

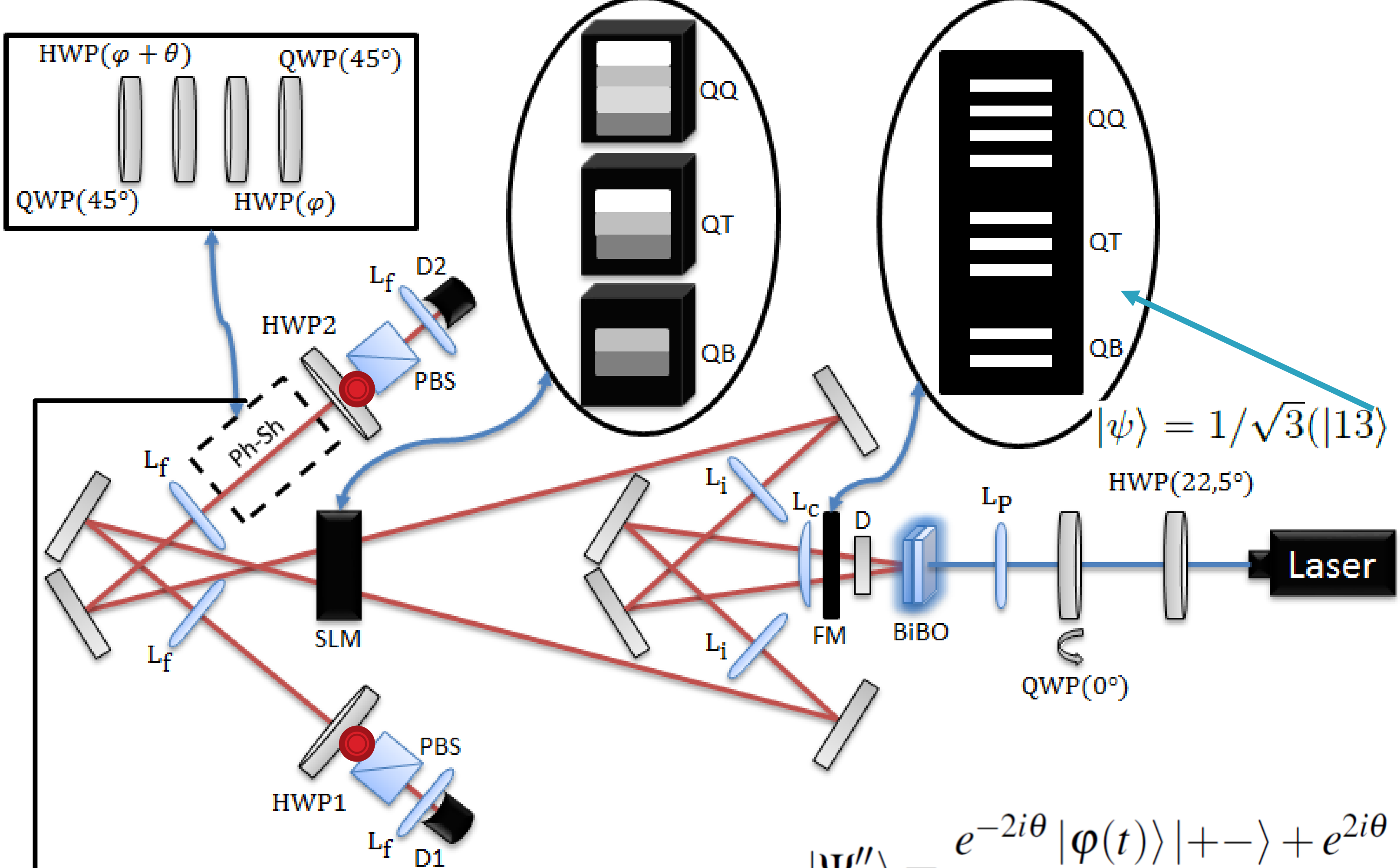
$$|\psi_0\rangle = \frac{|hh\rangle + e^{i\lambda}|vv\rangle}{\sqrt{2}} \otimes \frac{1}{\sqrt{d}} \sum_{m=1}^{d} |m, d-m+1\rangle$$

$$|\varphi(0)\rangle \otimes |hh\rangle \quad \rightarrow \quad |\varphi(t)\rangle \otimes |hh\rangle$$

$$|\varphi(0)\rangle \otimes |vv\rangle \quad \rightarrow \quad |\varphi(0)\rangle \otimes |vv\rangle$$

$$|\varphi(t)\rangle \quad = \quad U(t) \otimes V(t)\, |\varphi(0)\rangle$$

$$\boxed{|\Psi'\rangle = \frac{|\varphi(t)\rangle\,|hh\rangle + |\varphi(0)\rangle\,|vv\rangle}{\sqrt{2}}}$$



$$|\psi\rangle = 1/\sqrt{3}(|13\rangle + |22\rangle + |31\rangle)$$

$$|h\rangle \to e^{-2i\theta}|v\rangle \text{ and } |v\rangle \to e^{2i\theta}|h\rangle$$

$$|\Psi''\rangle = \frac{e^{-2i\theta}|\varphi(t)\rangle|+-\rangle + e^{2i\theta}|\varphi(0)\rangle|-+\rangle}{\sqrt{2}}$$

$$|\pm\rangle = (|h\rangle \pm |v\rangle)/\sqrt{2}$$

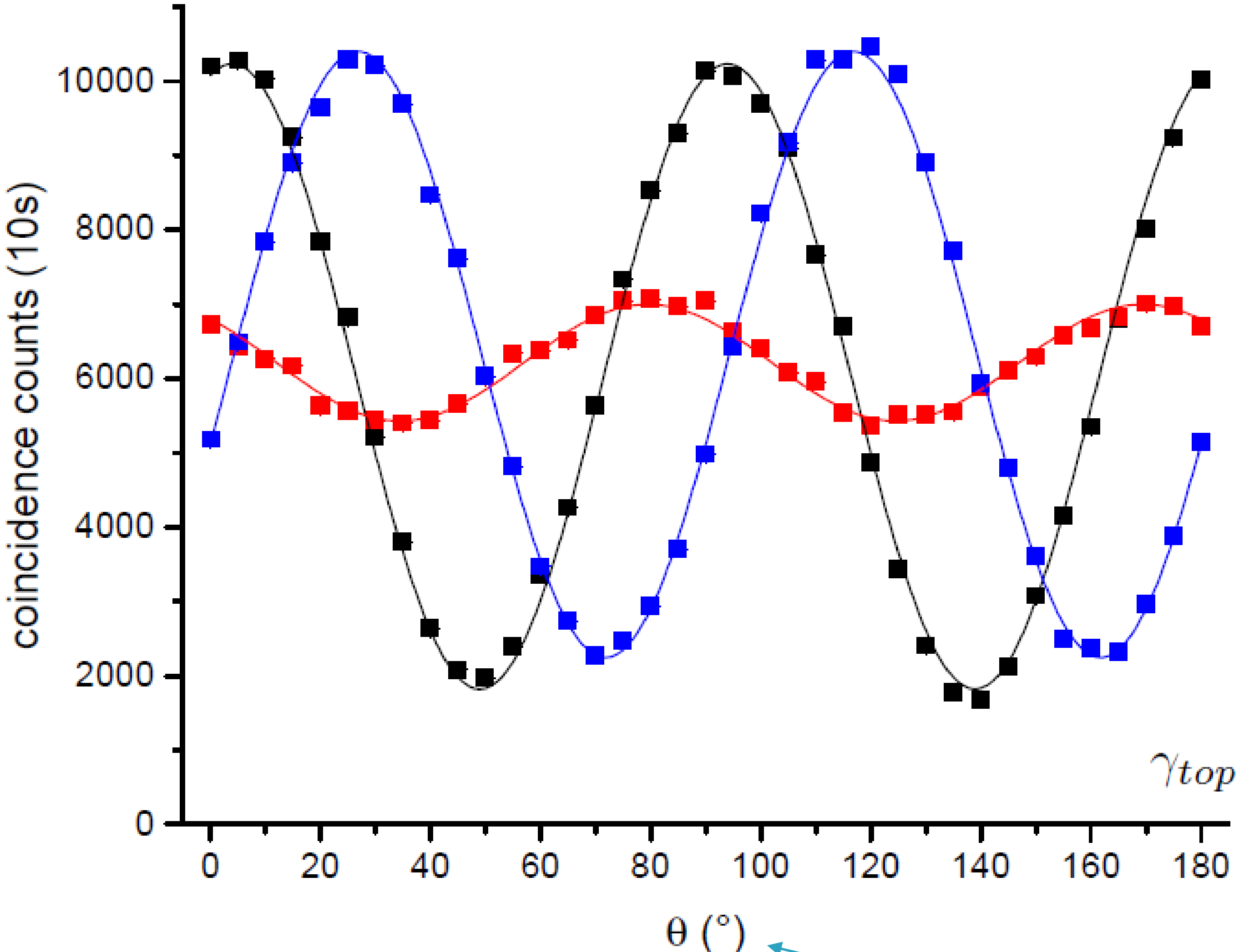
$$C = \frac{1}{2d}\sum_{m,n=1}^{d} \delta_{m,d-m+1} \sin^2\left(\frac{\xi_m + \chi_n + 4\theta}{2}\right)$$

No post-selection!

t= 0, $\frac{1}{2}$, 1

D = 4

$(94 \pm 3)^{\circ}$

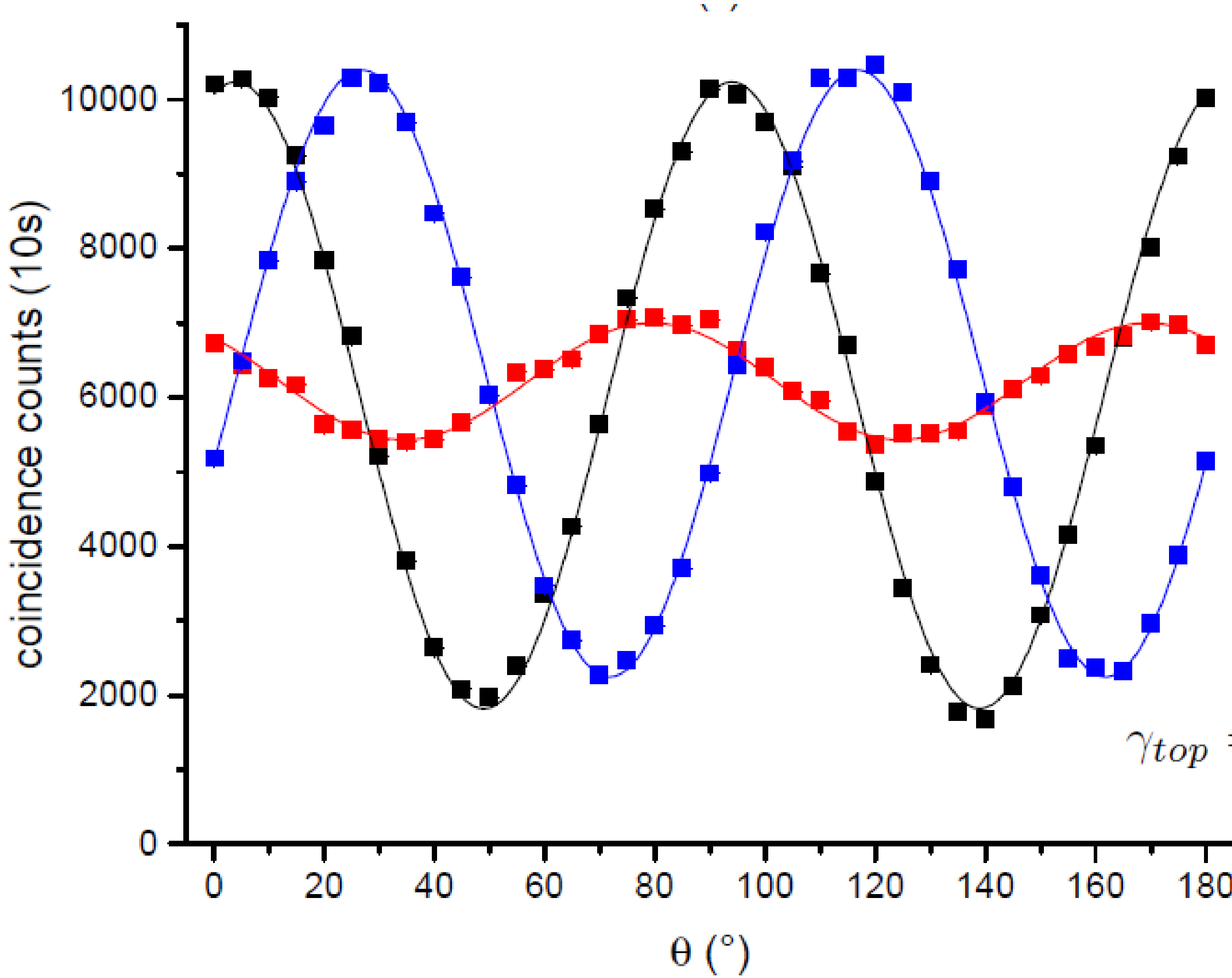


$\gamma_{top} = 2\pi/d.$

t= $0, \frac{1}{2}, 1$

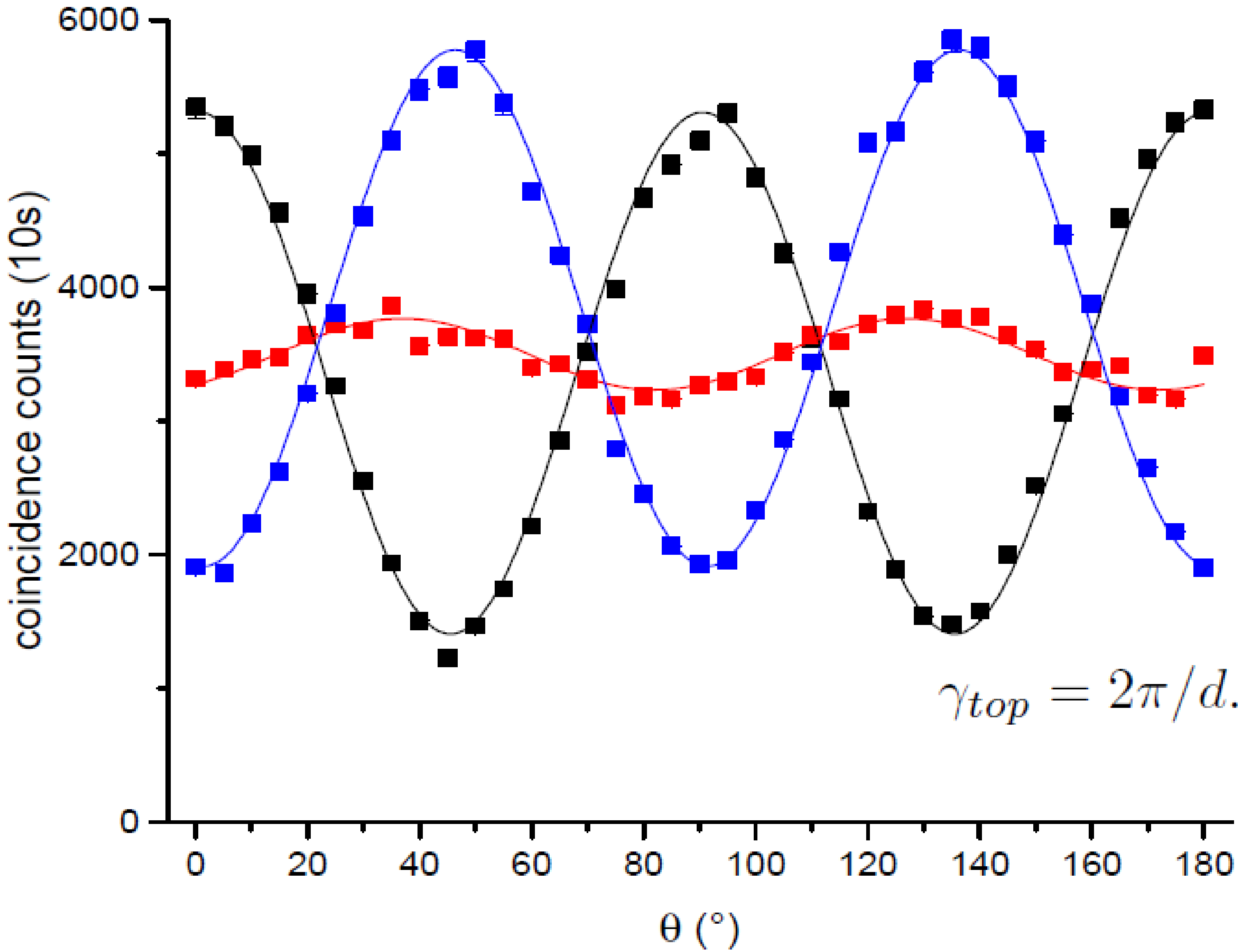
D = 3

$(126 \pm 3)^{o}$



$\gamma_{top} = 2\pi/d.$

t= $0, \frac{1}{2}, 1$ D = 2 $(182 \pm 7)^{\circ}$



Summary

- Qudits
- Fractional topological phase (FTF) for two qudits from the polar decomposition
- How to measure FTF through the coincidence counts of a entangled photon pair.
- Using Hyperentangled states.
- Noise added and robustness of the fractional topological phase.
- Conclusions

Robustness of the fractional topological phase to the phase noise

R. A. Ribeiro[1], A. A. Matoso[1], L. E. Oxman[2], A. Z. Khoury[2], and S. Pádua[1]
[1]Departamento de Física,
Universidade Federal de Minas Gerais,
31270-901 Belo Horizonte, Minas Gerais, Brazil
[2]Instituto de Física, Universidade Federal Fluminense,
24210-346 Niterói, Rio de Janeiro, Brazil

In this work we demonstrate theoretically via Kraus maps that the fractional topological phase in qudits is robust to phase noise (path dephasing). In our proposal, dephasing noise is inserted in an experimental optical setup proposed for measuring the fractional topological phase on prepared photonic spatially qudits after local unitary operations are applied. Qudits states can be prepared with photon pairs generated by spontaneous parametric down-conversion crossing a d-slit array.

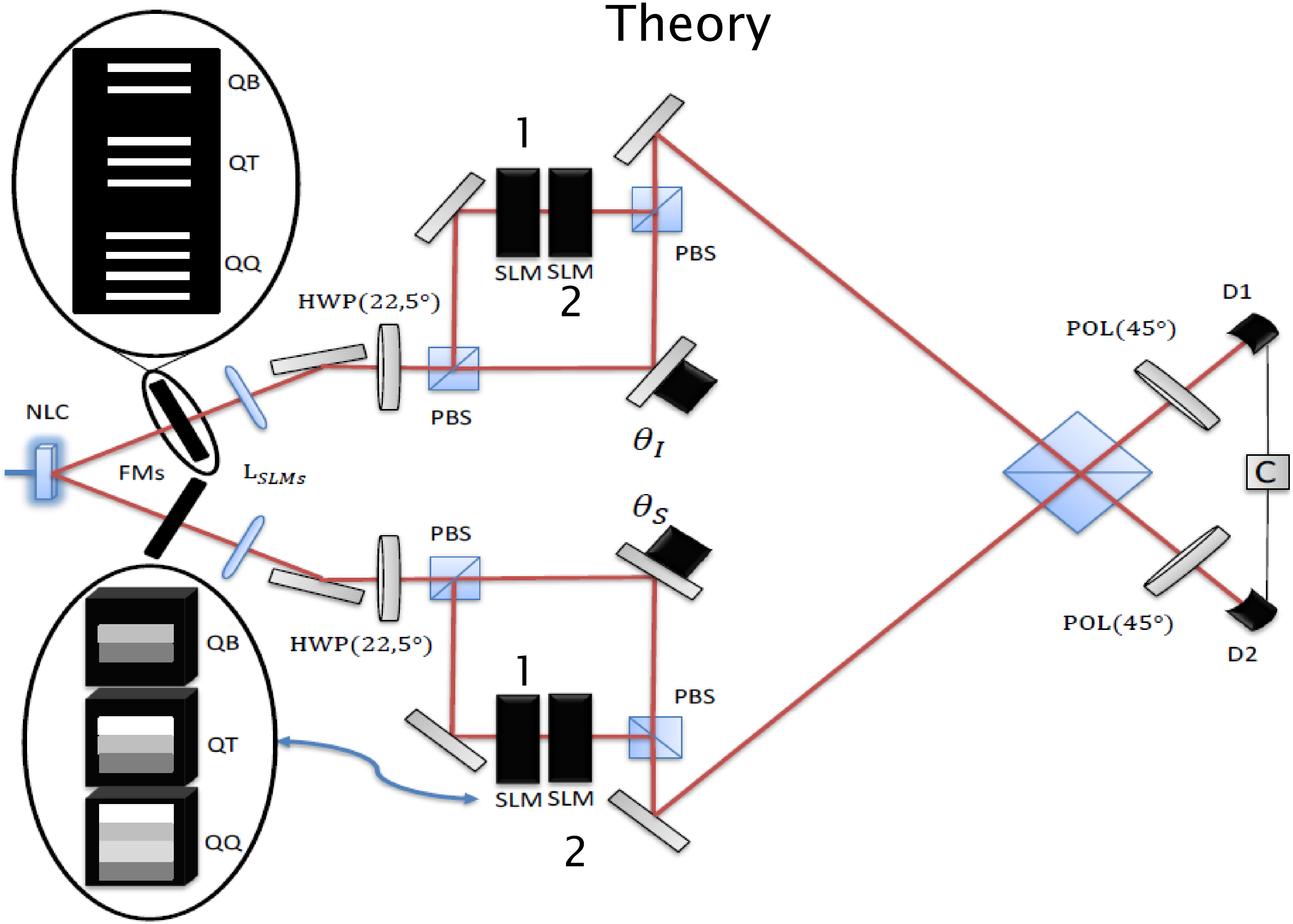
R. A. Ribeiro, A. A. Matoso, L. E. Oxman, A. Z. Khoury, and S. Pádua, Phys. Rev. A **99**, 042101 (2019)

Geometric phase associated with the evolution of a system subjected to decoherence.

A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **90**, 160402 (2003).

L. E. Oxman, A. Z. Khoury, F. C. Lombardo, and P. I. Villar, Annals of Physics **390**, 159 (2018).

Theory



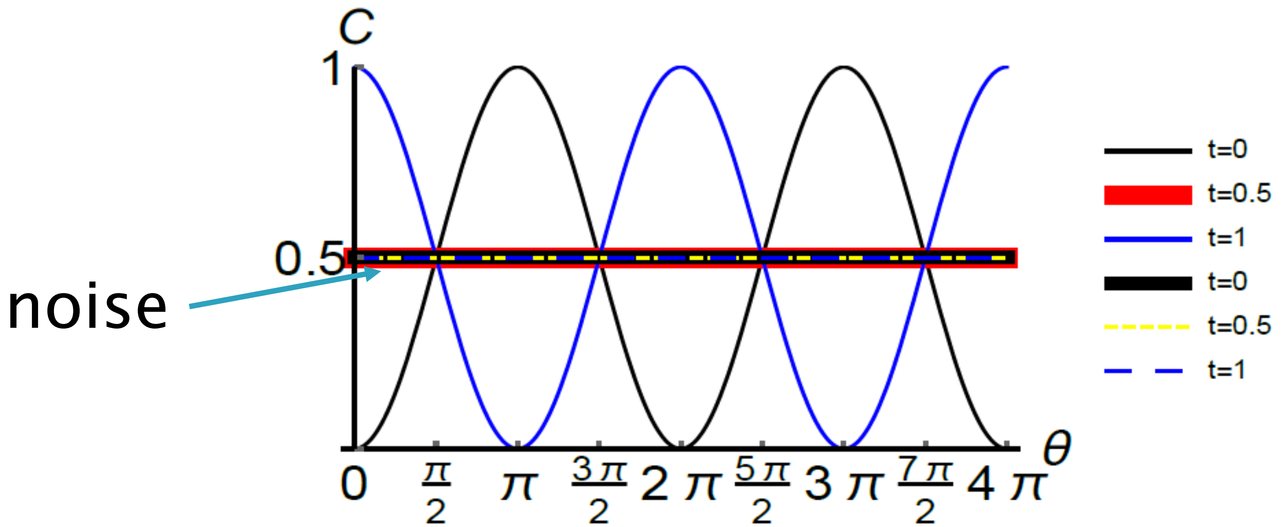
QB
QT
QQ
1
SLM SLM
2
PBS
HWP(22,5°)
PBS
NLC
FMs
L_{SLMs}
θ_I
θ_S
PBS
HWP(22,5°)
1
SLM SLM
2
PBS
D1
POL(45°)
C
POL(45°)
D2
QB
QT
QQ

FTP-Experiment under Noise Phase

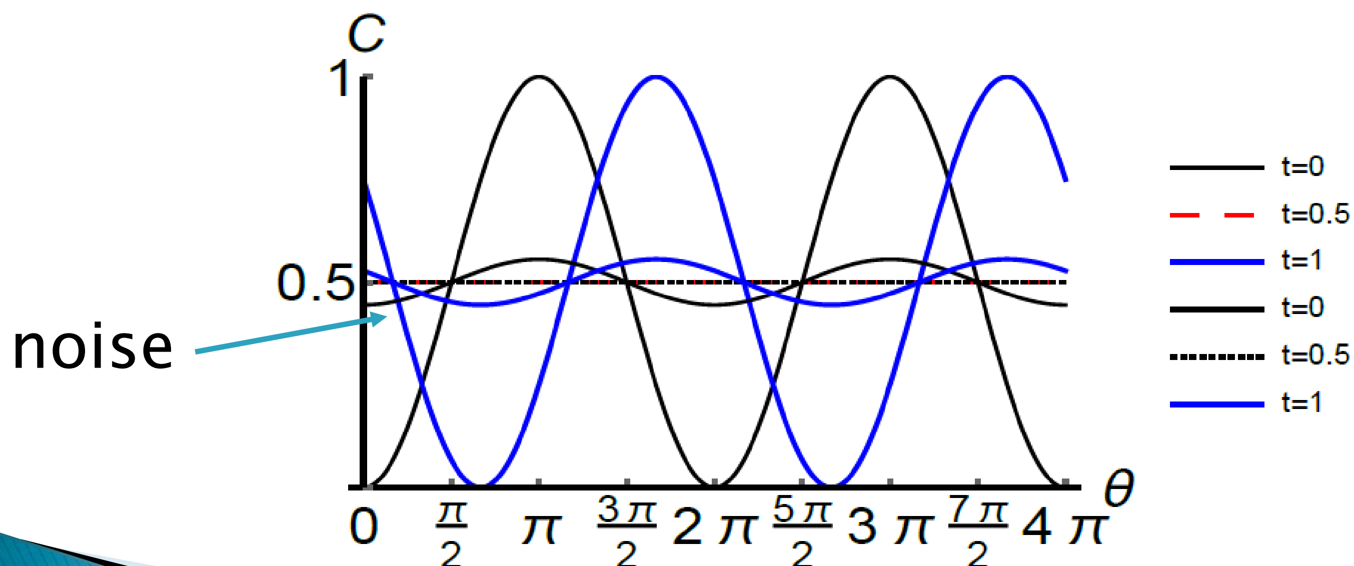
t	C_{me_p} of d=2	v_{me}
0	$\frac{1}{2}-\frac{(-1+p)^2}{2}Cos(\theta)$	$(-1+p)^2$
$\frac{1}{2}$	$\frac{1}{2}$	0
1	$\frac{1}{2}-\frac{(-1+p)^2}{2}Cos(\theta-\pi)$	$(-1+p)^2$

t	C_{me_p} of d=3	v_{me}
0	$\frac{1}{2}-\frac{(3-2p)^2}{18}Cos(\theta)$	$\frac{(3-2p)^2}{9}$
$\frac{1}{2}$	$\frac{1}{2}$	0
1	$\frac{1}{2}-\frac{(3-2p)^2}{18}Cos(\theta-\frac{2\pi}{3})$	$\frac{(3-2p)^2}{9}$

$p_{hj}=p_h\times p_j$, $p_{h\neq 0}=p_{j\neq 0}=p/d$ and $p_0=1-p$

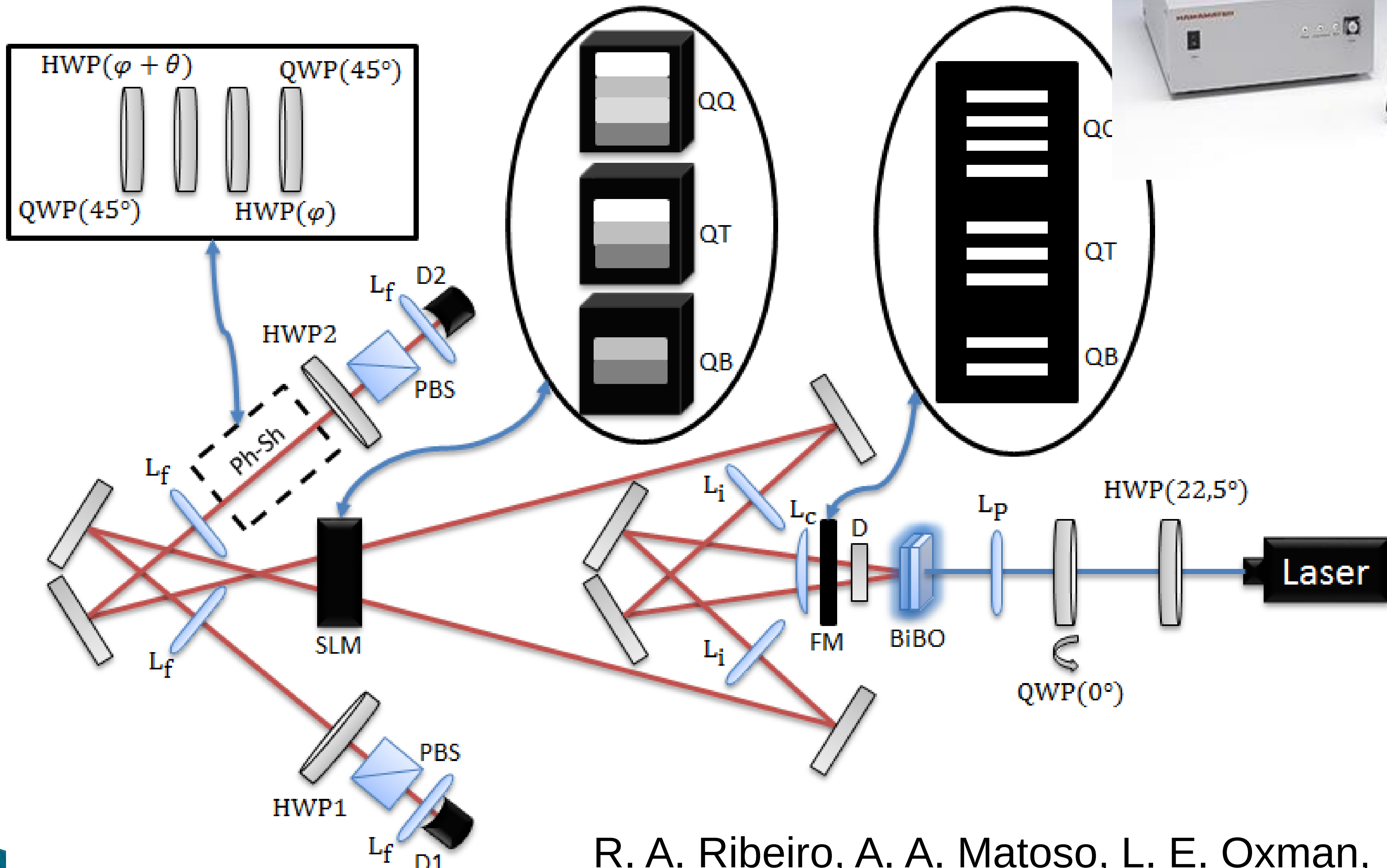


qubit

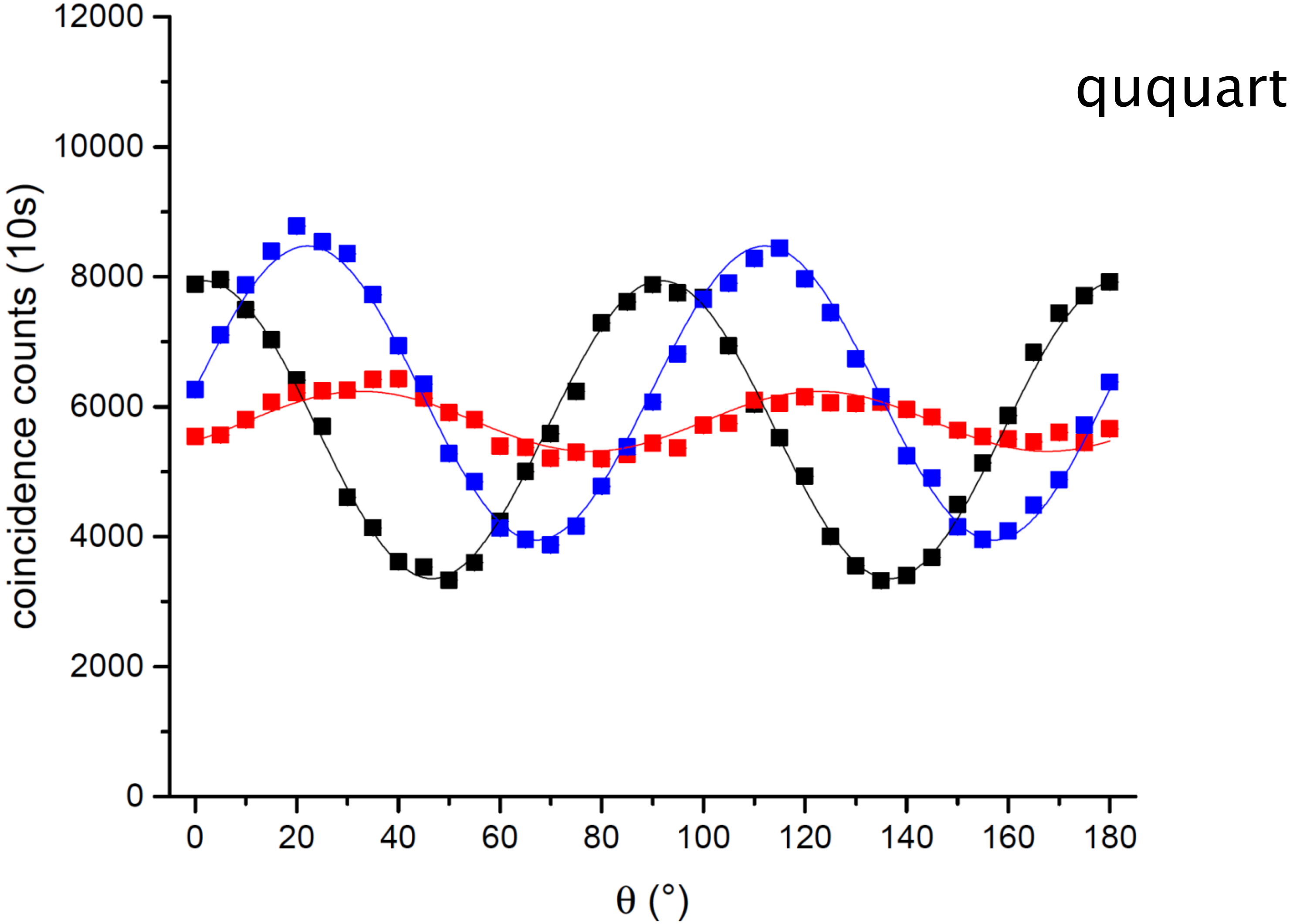


qutrit

Experimental Results

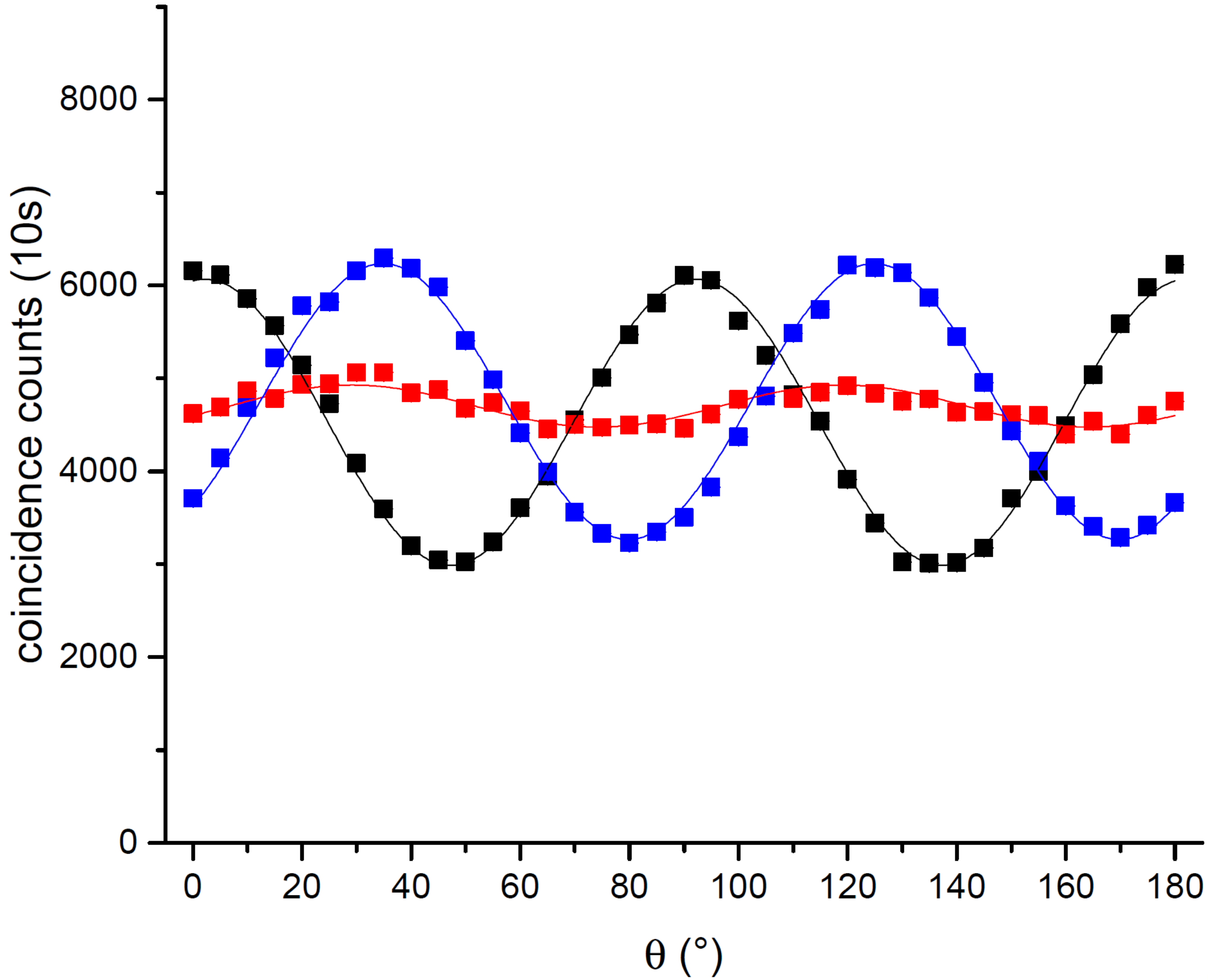


R. A. Ribeiro, A. A. Matoso, L. E. Oxman, A. Z. Khoury, and S. Pádua, unpublished (2019).

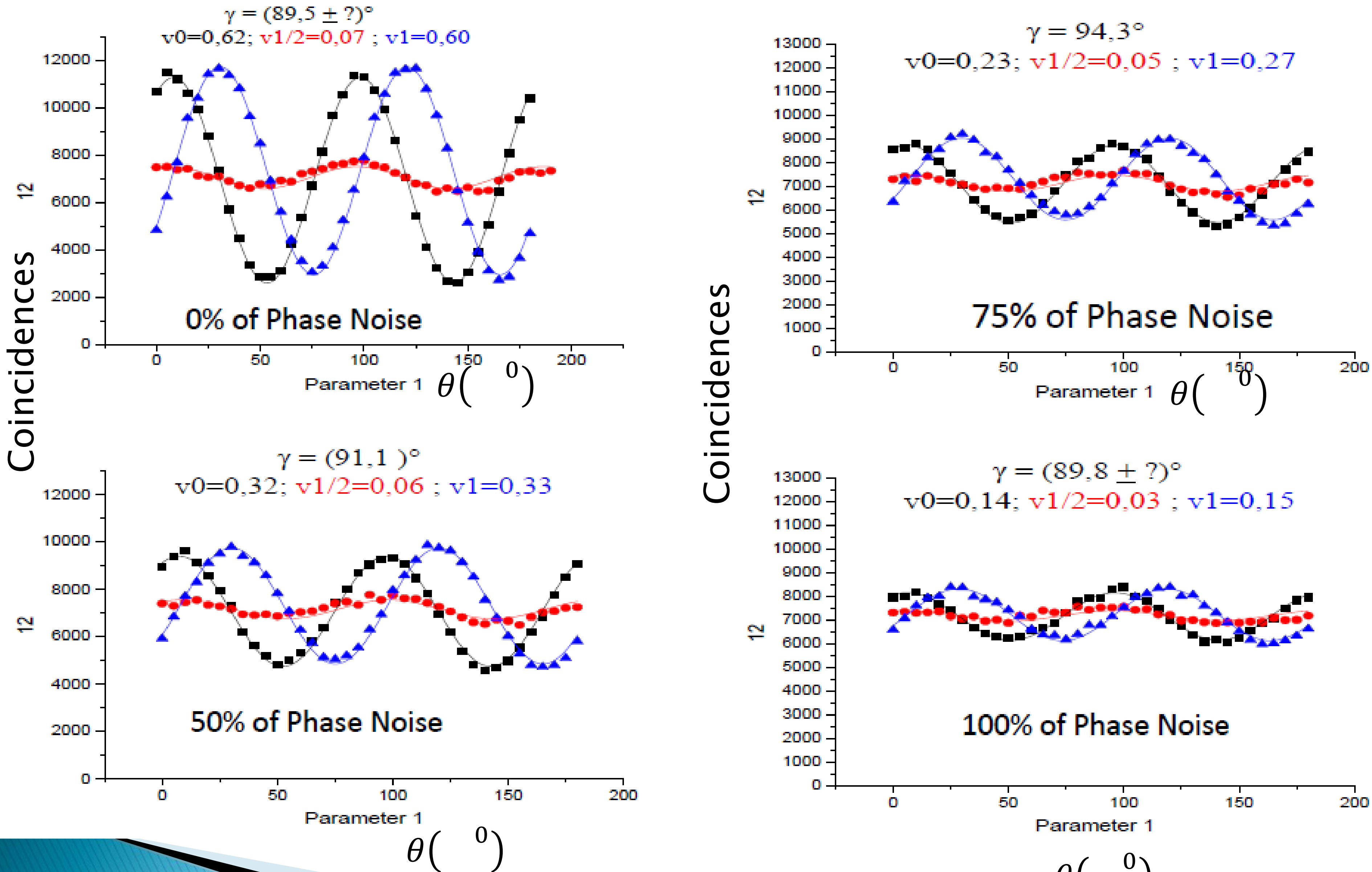


$$\gamma = (82 \pm 1)^{\circ}$$

qutrit



$$\gamma = (130 \pm 1)^\circ$$


γ = (89,5 ± ?)°
v0=0,62; v1/2=0,07 ; v1=0,60
0% of Phase Noise
Coincidences
12
Parameter 1
θ(0)
γ = 94,3°
v0=0,23; v1/2=0,05 ; v1=0,27
75% of Phase Noise
γ = (91,1)°
v0=0,32; v1/2=0,06 ; v1=0,33
50% of Phase Noise
γ = (89,8 ± ?)°
v0=0,14; v1/2=0,03 ; v1=0,15
100% of Phase Noise

ququarts

Conclusions

- A photonic interferometer is set to measure the FTP through the coincidence counts of a photon pair.

- The photons are encoded in path variables (slit states) and the polarization was an ancilla to implement the local operations using the SLMs.

- Measurement of the FTP for entangled qudit states with one local operation.

- It was demonstrated the dependence of the phase on the dimension d ($\frac{2l\pi}{d}$).

- Hyperentanglement is a resource that allows the measurement without path interferometers. No post selection. Better signal/noise.
- Roboustness of the topological phase is demonstrated theoretically and in preliminary measurements.

Thank you!

Why Topological?

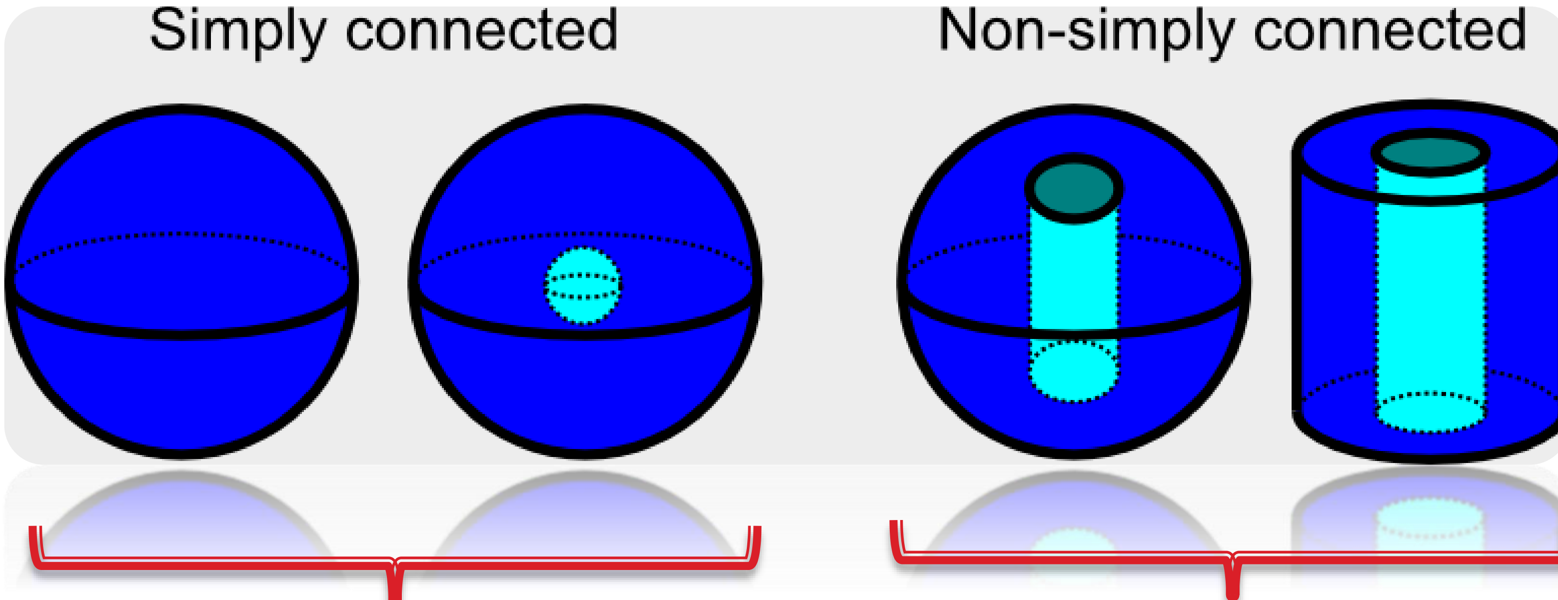
Using the polar decomposition and spliting γ for each sector of it:

$$e^{i\phi(T)}S(T)Q(T) = \left(e^{i\Delta\phi}e^{i\phi(0)}\right)\left(e^{i\gamma_s}S(0)\right)\left(e^{i\gamma_q}Q(0)\right)$$

From

can s

So, t



SU(d) matrices

SU(d) matrices with the identification

$$S(T) = e^{i\gamma_s}S(0)$$

Anholonomy is a geometric concept: the failure of some quantities to come back to their original values when others, which drive them, are forced to return. The failure derives from nonintegrability of the driving law. In this lecture I will concentrate on an example of

M. Berry

Moreover, when the qudits are operated by local SU(d) transformations, the U(1) sector remains stationary and the fractional phases are the only ones attainable under cyclic evolutions. These fractional phases are from topological nature since the set $\{S \in \mathrm{SU}(d) | e^{i\gamma}S \equiv S\}$ is not simply connected and different homotopy classes of closed trajectories can be conceived. A detailed analysis for qubits can be found in refs[8,9].

8. Milman, P. & Mosseri, R. Topological phase for entangled two-qubit states. *Phys. Rev. Lett.* **90**, 230403, https://doi.org/10.1103/PhysRevLett.90.230403 (2003).
9. Milman, P. Phase dynamics of entangled qubits. *Phys. Rev. A* **73**, 062118, https://doi.org/10.1103/PhysRevA.73.062118 (2006).

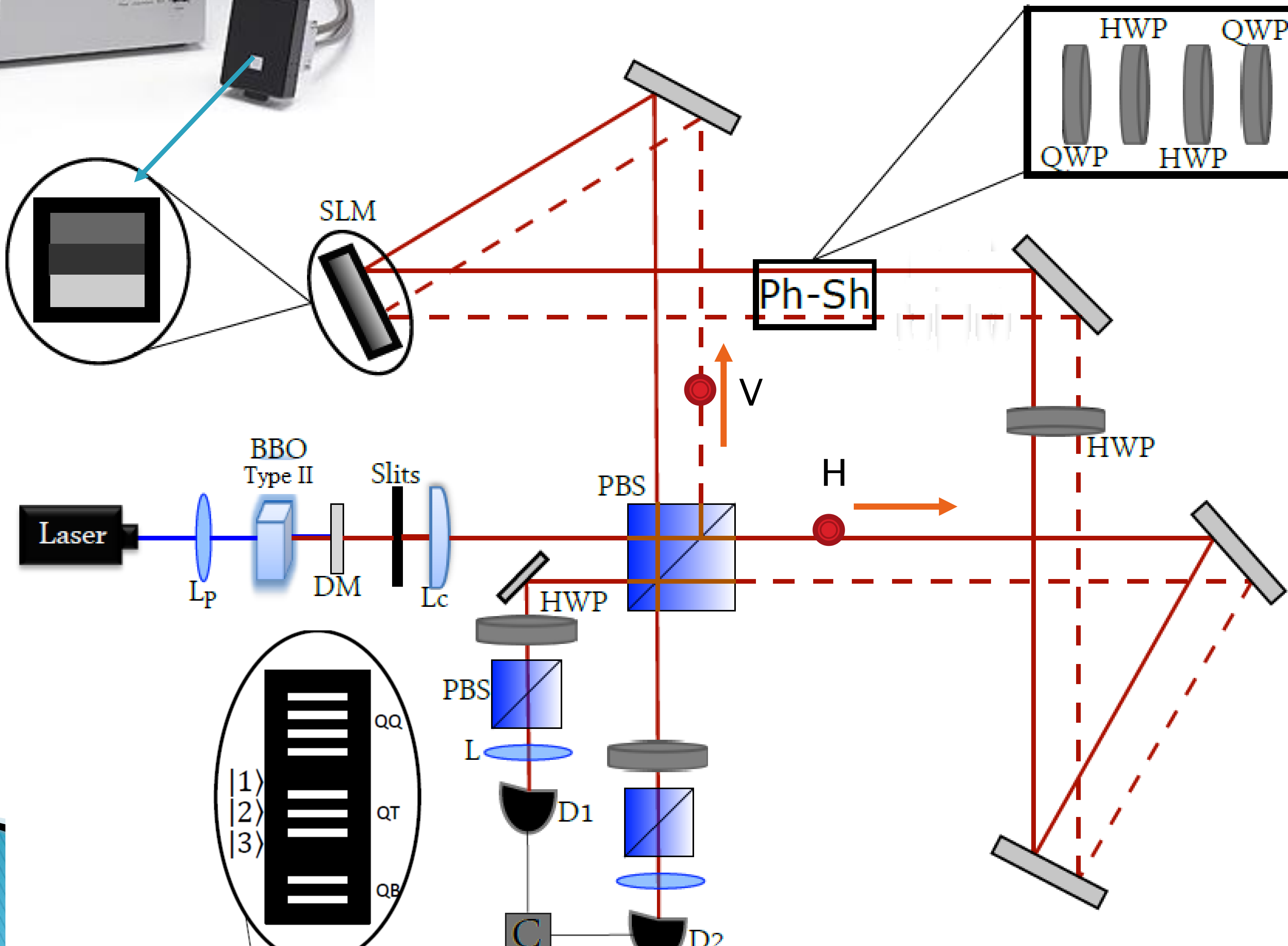
Take a pencil, lay it on the north pole of a globe and point it in the direction of any of the meridians: the lines of longitude that radiate from the pole. Move the pencil down along the line to the equator and, keeping it perpendicular to the equator, slide it to another line of longitude. Move the pencil back to the north pole along the new meridian, and you will find that although the pencil has been returned to its starting point and at no time was rotated, it no longer points along the original line of longitude.

This simple exercise illustrates how the "parallel transport" of a vector (a quantity that has both length and direction) around a circuit on a curved surface results in an anholonomy: the failure of certain variables describing the system to return to their original values. The anholonomy in the example results from the fact that the pencil was forced to trace out a circuit on the surface of a sphere while remaining parallel to the meridians at all times. It is a purely geometric phenomenon; it does not depend on the energy or mass of the pencil. Moreover, it does not depend on the pencil's initial direction. The extent of the anholonomy depends solely on the area and curvature of the surface enclosed by the circuit.

In 1983 I found that a similar geometric effect exists in the quantum waves that describe matter and its interactions on the smallest scales. In this case the anholonomy appears in a system's wave function (the mathematical description of a system's physical state) after the system has been transported around a cyclic circuit on an abstract surface in "parameter space." I call this anholonomy the geometric phase, because it manifests itself specifically as a shift in the wave function's phase: a quantity that describes where the wave function is in its oscillatory cycle at any given time and place.

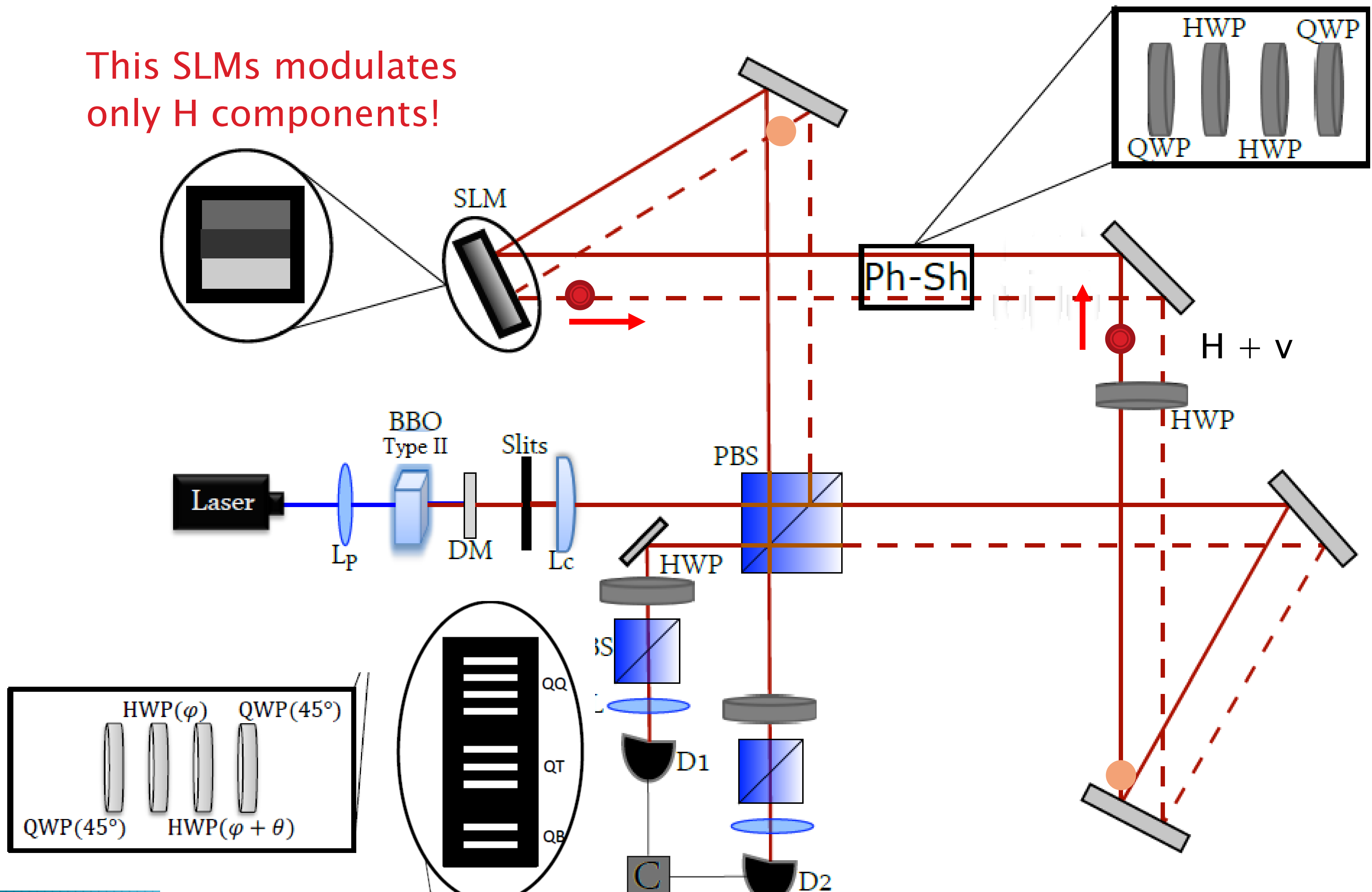
Berry, Scientific American1988

measure the FTF

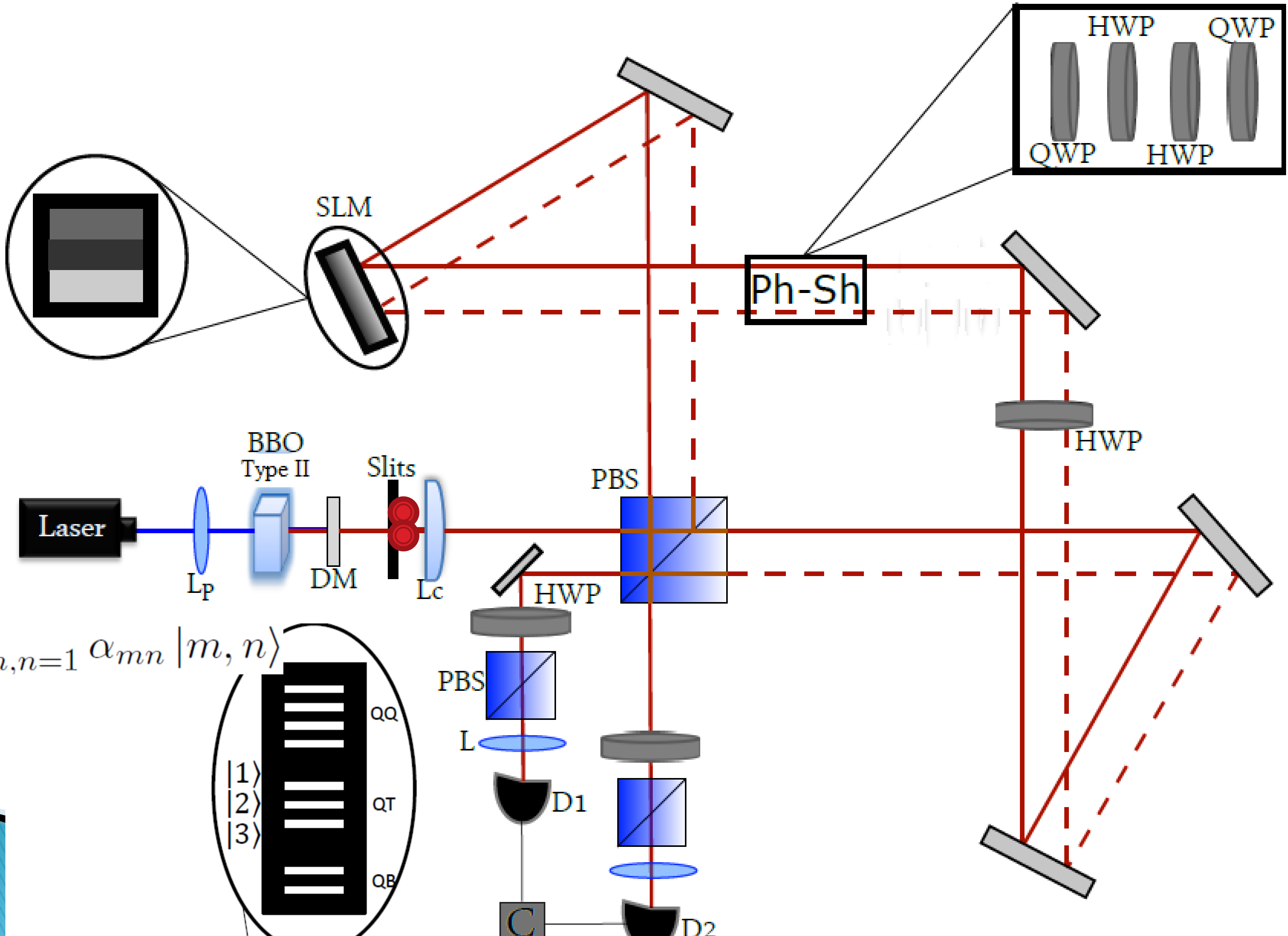


how to measure the FTF

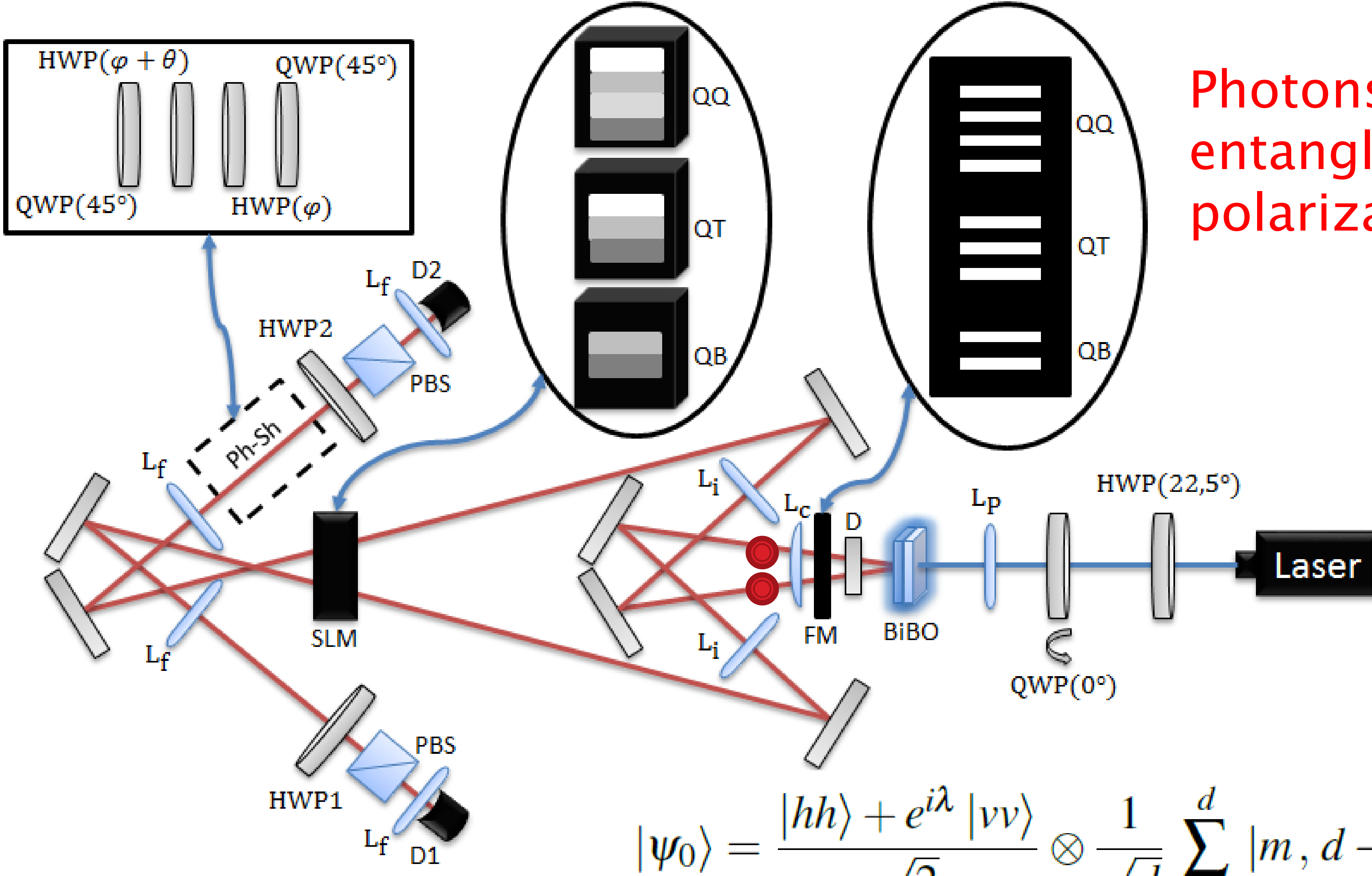
This SLMs modulates only H components!



how to measure the FTF



$$|\psi\rangle = \sum_{m,n=1}^{d} \alpha_{mn} |m, n\rangle$$



Photons entangled in polarization

$$|\psi_0\rangle = \frac{|hh\rangle + e^{i\lambda}|vv\rangle}{\sqrt{2}} \otimes \frac{1}{\sqrt{d}} \sum_{m=1}^{d} |m\,, d-m+1\rangle$$

Path correlation