QUANTUM ENTANGLEMENT IN BASE & TARGET SPACE

SUMIT R. DAS

UNIVERSITY OF KENTUCKY
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

(S.R.D, A. Kaushal, G. Mandal & S. Trivedi
2004.0061

S.R.D, A. Kaushal, S. Li, G. Mandal & S. Trivedi
2010.XXXX)

Saturday, September 19, 2020 3:40 PM

ENTANGLEMENT IS PERAHPS THE MOST UNIQUE FEATURE OF QUANTUM MECHANICS

CONSIDER E.G. TWO PARTICLES. IN SOME EXTERNAL POTENTIAL

$$H = \frac{1}{2m}\left(P_1^2 + P_2^2\right) + \frac{1}{2}m\omega^2\left(x_1^2 + x_2^2\right)$$

THEY DO NOT INTERACT WITH EACH OTHER – THEY ONLY FEEL THE SAME POTENTIAL.

THIS MEANS THAT THE MOST GENERAL WAVEFUNCTION IS OF THE FORM

$$\psi(x_1, x_2) = \sum_{mn} C_{mn}\, u_m(x_1)\, u_n(x_2)$$

WHERE $u_n(x)$ DENOTE WAVEFUNCTION OF A SINGLE PARTICLE. FOR e.g.

$$u_0(x) = \left(\frac{m\omega}{\pi}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

$$u_1(x) = \frac{1}{\sqrt{2}} \times \left(\frac{m\omega}{\pi}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

CONSIDER NOW 2 POSSIBLE WAVEFUNCTIONS OF THE 2 PARTICLE SYSTEM

$$\psi_1(x_1, x_2) = u_0(x_1)\, u_1(x_2).$$

$$\psi_2(x_1, x_2) = \frac{1}{\sqrt{2}}\left(u_0(x_1)\, u_1(x_2) - u_1(x_0)\, u_0(x_2)\right)$$

BOTH ARE VALID WAVEFUNCTIONS IF THE PARTICLES ARE DISTINGUISHABLE

HOWEVER IF THEY ARE IDENTICAL ONLY ψ_2 IS A VALID WAVEFUNCTION – THIS IS WAVEFUNCTION IF THE PARTICLES ARE FERMIONS.

THERE IS A FUNDAMENTAL DIFFERENCE BETWEEN THESE WAVEFUNCTIONS

TO SEE THIS CONSIDER MEASURING THE QUANTITY x_1^2

FOR THE FIRST WAVEFUNCTION

$$\begin{aligned}
\langle x_1^2 \rangle &= \int dx_1 dx_2\, \psi_1^*(x_1, x_2)\, x_1^2\, \psi_1(x_1, x_2) \\
&= \int dx_1 dx_2\, u_0^*(x_1)\, u_1^*(x_2)\, x_1^2\, u_0(x_1) u_1(x_2) \\
&= \left[\int dx_1\, u_0^*(x_1)\, x_1^2\, u_0(x_1)\right] \left[\int dx_2\, u_1^*(x_2) u_1^*(x_2)\right] \\
&= \int dx_1\, u_0^*(x_1)\, x_1^2\, u_0(x_1)
\end{aligned}$$

THIS IS EXACTLY THE ANSWER YOU WOULD OBTAIN EVEN IF THE SECOND PARTICLE WAS NOT THERE

Saturday, September 19, 2020 5:04 PM

FOR THE SECOND WAVEFUNCTION

$$\langle x_1^2 \rangle = \frac{1}{2}\int dx_1\, u_0^*(x_1)\, x_1^2\, u_0(x_1) + \frac{1}{2}\int dx_1\, u_1^*(x_1)\, x_1^2\, u_1(x_1)$$

SO FAR AS THE FIRST PARTICLE IS CONCERNED

THIS IS NOT THE EXPECTATION VALUE IN ANY STATE

RATHER THIS IS SUM OF EXPECTATION VALUES WEIGHTED BY PROBABILITY OF BEING IN THAT STATE

THIS IS AS IF THIS PARTICLE IS IN A MIXED STATE

MEASUREMENTS OF PROPERTIES OF PARTICLE 1 NOT INDEPENDENT OF PARTICLE 2

WAVEFUNCTIONS LIKE

$$\frac{1}{\sqrt{2}}\left[u_0(x_1)u_1(x_2) - u_1(x_1)u_0(x_2)\right]$$

EXAMPLE OF AN ENTANGLED STATE

IF YOU ONLY MEASURE OBSERVABLES WHICH RELATE TO THE FIRST PARTICLE — YOU WILL THINK THAT THE STATE IS MIXED.

MIXED STATE: NO COMPLETE KNOWLEDGE OF THE STATE

P_α: PROBABILITY OF SYSTEM TO BE IN A PATRICULAR STATE.

$\langle \mathcal{O} \rangle_\alpha$: EXPECTATION VALUE OF OBSERVABLE

$$\langle \mathcal{O} \rangle = \sum_\alpha P_\alpha \langle \mathcal{O} \rangle_\alpha$$

VON NEUMANN ENTROPY

$$S = -\sum_{\alpha} P_\alpha \log P_\alpha$$

IF THE MIXED NATURE COMES ONLY FROM QUANTUM ENTANGLEMENT

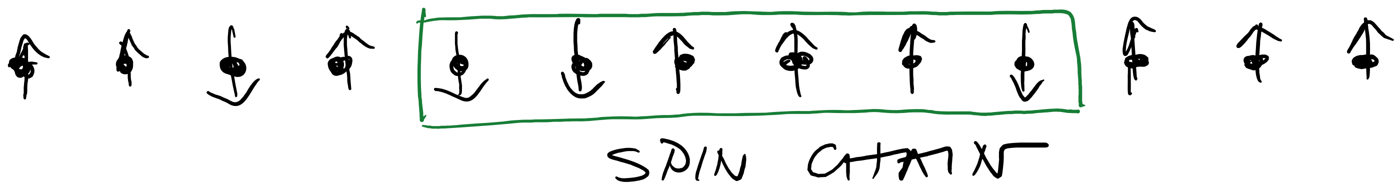
— ENTANGLEMENT ENTROPY

PURE STATE : $P_\alpha = 1$ FOR ONE α

$P_\alpha = 0$ FOR ALL OTHER α

$S = 0$

GEOMETRIC ENTROPY



SUPPOSE WE MAKE A MEASUREMENT ONLY ON THIS SUBSET OF SPINS

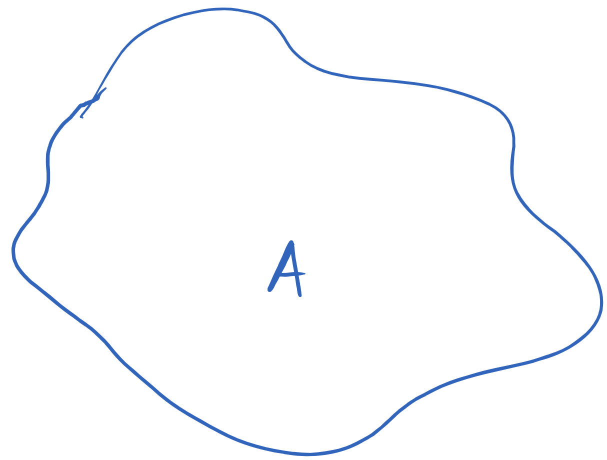
— EVEN IF THE WHOLE SYSTEM IS IN A PURE STATE WHICH IS ENTANGLED.

THEN THESE MEASUREMENTS CAN BE OBTAINED ENTIRELY IN TERMS OF THESE SMALLER NUMBER OF SPINS

— HOWEVER THIS WOULD BE A MIXED STATE.

ENTANGLEMENT ENTROPY IS A MEASURE OF THIS

SIMILARLY IN QUANTUM FIELD THEORY IF DEGREES OF FREEDOM IN A ARE ENTANGLED WITH THE REST



IN VACUUM

$S(A) \propto$ Area of the boundary

(log PIECES IN 1+1 DIMENSION - OTHER CORRECTIONS)

$\boxed{\text{BASE SPACE ENTANGLEMENT}}$

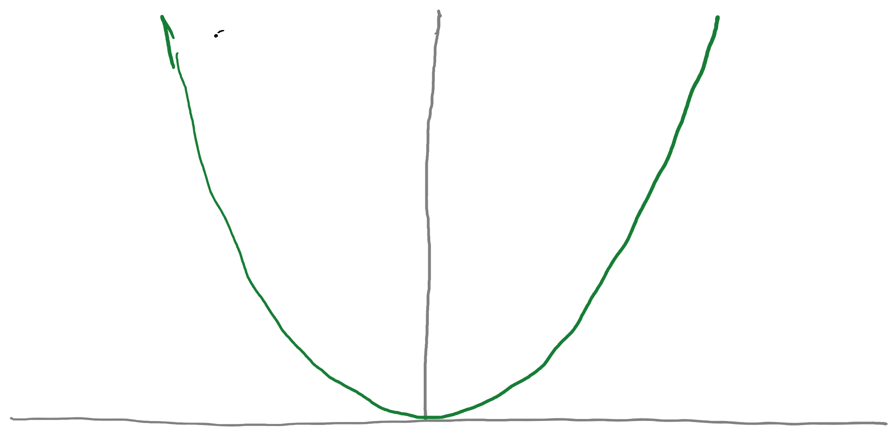
ASSOCIATED WITH A PARTITION OF SPACE

Saturday, September 19, 2020 9:19 PM

TARGET SPACE ENTANGLEMENT

CONSIDER NOW A SINGLE PARTICLE IN SOME POTENTIAL e.g. HO

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$



SUPPOSE WE CAN MEASURE PARTICLE'S POSITION MOMENTUM etc SO LONG AS THE PARTICLE IS IN THE $x > 0$ REGION

Monday, September 21, 2020 7:09 PM

HERE IS ONE WAY YOU CAN DO THIS.
CONSIDER THE OPERATOR

$$\mathcal{O} = \int_0^\infty dy\, \delta(y - \hat{x})\, \hat{x}^2$$

IN SOME STATE WITH WAVEFUNCTION $\psi(x)$

$$\langle \psi | \mathcal{O} | \psi \rangle = \int_0^\infty dx\, \psi^*(x)\, x^2 \psi(x)$$

SO FAR AS THE $x > 0$ REGION IS CONCERNED
THIS IS NOT EXPECTATION VALUE IN A
PURE STATE SINCE

$$\int_0^\infty dx\, \psi^*(x) \psi(x) \neq 1$$

RATHER

$$P = \int_0^\infty dx\, \psi^*(x) \psi(x) =$$ PROBABILITY THAT PARTICLE IS IN $x > 0$ REGION

IN FACT IF YOU ARE RESTRICTED TO SUCH MEASUREMENTS YOU WILL CONCLUDE THAT THE PARTICLE IS IN A MIXED STATE.

THERE IS AN ASSOCIATED VON NEUMANN ENTROPY

$$S = -p\log p - (1-p)\log(1-p).$$

THE COORDINATE X IS TARGET SPACE VARIABLE

THIS IS THE SIMPLEST EXAMPLE OF

TARGET SPACE ENTANGLEMENT

INSTEAD OF $X>0$ WE COULD RESTRICT TO ANY REGION OF REAL LINE

IN A GENERIC STATE THE REGION $x>0$
IS ENTANGLED WITH THE REGION $x<0$

– BUT IN A RATHER DIFFERNT SENSE

WHEN IS THE STATE NOT ENTANGLED ?

THIS REQUIRES $\psi(x) = 0$ EVERYWHERE
OUTSIDE THE REGION OF INTEREST
($x>0$ IN THIS CASE)

THEN $\rho^2 = 1$ AND ENTROPY VANISHES

q`

Wednesday, September 23, 2020 1:02 AM

IF WE HAD 2 FERMIONS IN THIS POTENTIAL AN OPERATOR WHICH RESTRICTS TO e.g. $x>0$ REGION

$$\mathcal{O} = \int_0^\infty dy\, \delta(y-\hat{x}_1)\,\hat{x}_1^n + \int_0^\infty dy\, \delta(y-\hat{x}_2)\,\hat{x}_2^n$$

EXPECTATION VALUE IS A SUM OF SECTORS

$$\langle\psi|\mathcal{O}|\psi\rangle = \int_0^\infty dx_1 \int_0^\infty dx_2\, \psi^*(x_1,x_2)\,(x_1^n + x_2^n)\,\psi(x_1,x_2)$$

$$+\ 2\int_0^\infty dx_1 \int_{-\infty}^0 dy_2\, \psi^*(x_1,y_2)\, x_1^n\, \psi(x_1,y_2)$$

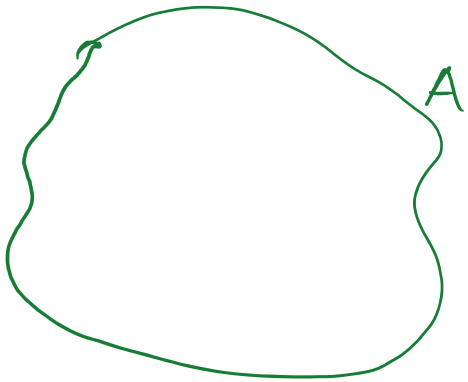
SIMILARLY FOR N PARTICLES

FORMALISM OF TARGET SPACE ENTANGLEMENT DEVELOPED IN (S.R.D, G. Mandal & S. Trivedi; E. Mazenc & Ranard)

Wednesday, September 23, 2020 1:16 AM

GEOMETRIC ENTANGLEMENT IN A THEORY OF GRAVITY

IN USUAL QUANTUM FIELD THEORY



$$S_{EE} = \frac{A}{\varepsilon^{d-1}}$$

ε : UV CUTOFF

IF WE HAVE DYNAMICAL GRAVITY IT IS TRICKY TO DEFINE SUCH A QUANTITY

- IN GRAVITY STRICTLY SPEAKING NO LOCAL OBSERVABLES
- IF METRIC IS FLUCTUATING TRICKY TO SPECIFY A SPATIAL REGION

HOWEVER, IN A REGIME WHERE QUANTUM GRAVITY CAN BE CONSIDERED A FIELD THEORY OF GRAVITONS THERE SHOULD BE AN APPROXIMATE NOTION

WHAT IS THE PRECISE MEANING OF SPATIAL ENTANGLEMENT REDUCES TO THIS APPROXIMATE NOTION IN AN APPROPRIATE REGIME ?

IF THERE IS SUCH A NOTION IN A PROPER THEORY OF GRAVITY LIKE STRING THEORY ENTANGLEMENT ENTROPY SHOULD BE FINITE

WHAT IS THE UV CUTOFF WHICH AUTOMATICALLY APPEARS ?

IS IT THE STRING LENGTH l_s ?

IS IT NEWTON'S CONSTANT G_N ?

PROPOSAL: IN GRAVITATIONAL THEORIES WHICH HAVE DUAL DESCRIPTIONS IN TERMS OF D-BRANES

THE PRECISE NOTION OF ENTANGLEMENT IN SPACE IS PROVIDED BY TARGET SPACE ENTANGLEMENT IN THE THEORY OF D-BRANES

DO BRANES

THESE ARE PARTICLE LIKE OBJECTS IN STRING THEORY

HOWEVER THEIR DYNAMICS IS DESCRIBED BY QUANTUM MECHANICS OF MATRICES

$$(X^I)_{ij} \qquad I = 1 \ldots\ldots 9$$

$$i,j = 1 \ldots\ldots N$$

HAMILTONIAN

$$H = \Lambda \, \mathrm{Tr} \left\{ \sum_{I=1}^{9} (\hat{P}^I)^2 + \sum_{I \neq J = 1}^{9} [X^I, X^J]^2 \right\}$$

YOU MAY THINK OF

$$(P^I)_{ij} \rightarrow \frac{\partial}{\partial X^I_{ij}}$$

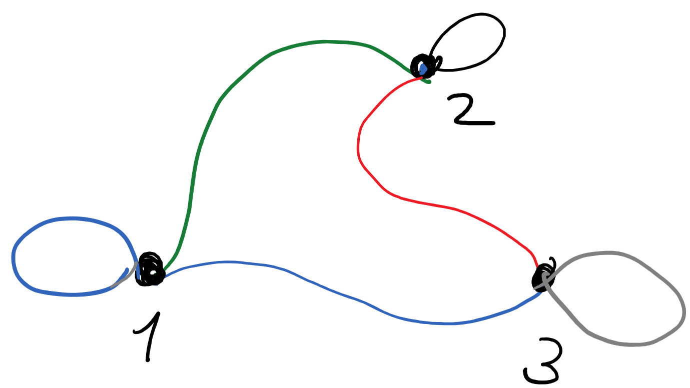
IF IN A STATE THE MATRICES COMMUTE
WE CAN SIMULTANEOUSLY DIAGONALIZE

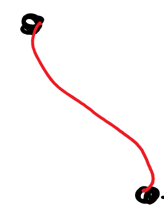
$$X^I_{..} \to \mathrm{diag}\left(x^I_1, x^I_2 \ldots x^I_N\right).$$

WE HAVE N FREE PARTICLES MOVING
IN 9 DIMENSIONS

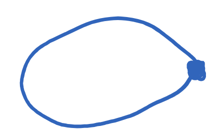
HOWEVER, GENERALLY THE MATRICES DO
NOT COMMUTE – WE THEN HAVE
PARTICLES JOINED BY STRINGS

$$\begin{pmatrix} X^I_{11} & X^I_{12} & X^I_{13} \\ X^I_{21} & X^I_{22} & X^I_{23} \\ X^I_{31} & X^I_{32} & X^I_{33} \end{pmatrix}$$





X^I_{23}, X^I_{32}



X^I_{11}

Wednesday, September 23, 2020 1:52 AM

IN FACT N D0 BRANES CAN FORM A BOUND STATE – THERE IS A DUAL DESCRIPTION OF THIS AS A NONTRIVIAL GRAVITY BACKGROUND IN 9+1 DIMENSIONS

WHEN $N \gg 1$ THIS GRAVITY BACKGROUND IS SMOOTH – AND FLUCTUATIONS INCLUDE GRAVITONS

9+1 DIMENSIONAL GRAVITATIONAL PHYSICS IN THIS BACKGROUND CAN BE DESCRIBED ENTIRELY IN TERMS OF THE 0+1 DIMENSIONAL QUANTUM MECHANICS

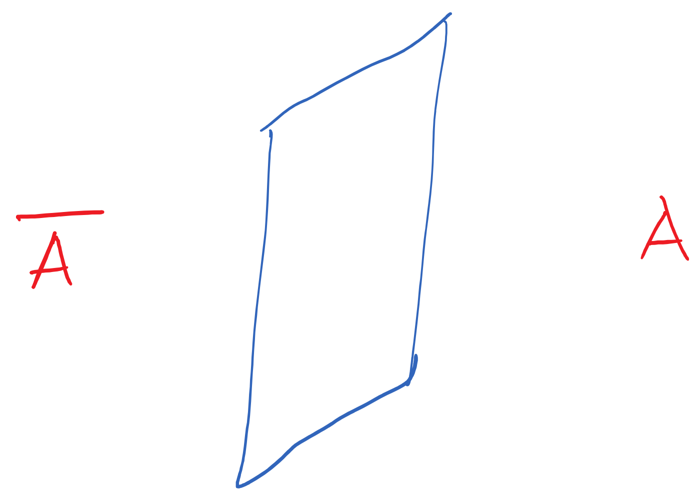
$\boxed{\text{HOLOGRAPHY}}$

(Itzhaki, Maldacena, Sonnenschein, Yankelowicz)

THERE IS A SLIGHTLY DIFFERENT REGIME WHERE THIS QUANTUM MECHANICS DESCRIBES M THEORY IN 10+1 DIMENSIONS

(Banks, Fischler, Shenker, Susskind)

SUPPOSE WE DIVIDE THE 9 SPACE DIMENSIONS BY A 8 DIMENSIONAL SURFACE



IF THE BACKGROUND WERE FIXED AND WE HAD QUANTUM FIELDS WE WOULD KNOW HOW TO TALK ABOUT THE ENTANGLEMENT OF A AND $\bar{A}$

IN THE PRESENCE OF DYNAMICAL GRAVITY THIS BECOMES CONFUSING

Wednesday, September 23, 2020 4:40 AM

BUT NOW WE KNOW THAT THIS 9+1 DIMENSIONAL SPACE IS MANUFACTURED FROM 9 NxN MATRICES

THE BASE SPACE OF THE GRAVITY THEORY IS THE TARGET SPACE OF THE MATRIX QUANTUM MECHANICS

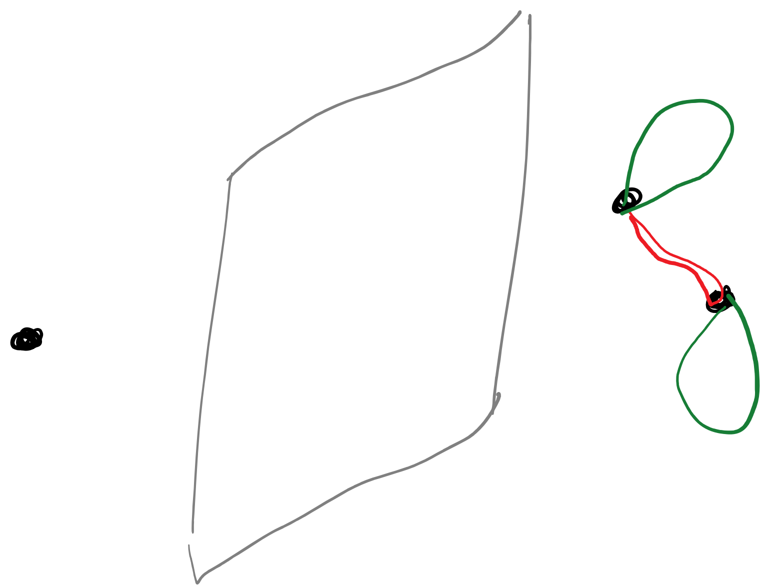
$\Rightarrow$ THE ENTANGLEMENT BETWEEN REGIONS OF THIS 9 DIMENSIONAL SPACE IS TARGET SPACE ENTANGLEMENT IN MATRIX QUANTUM MECHANICS

NOTIONS OF ENTANGLEMENT IN THE TARGET SPACE OF THIS THEORY CAN BE DEFINED IN A PRECISE WAY

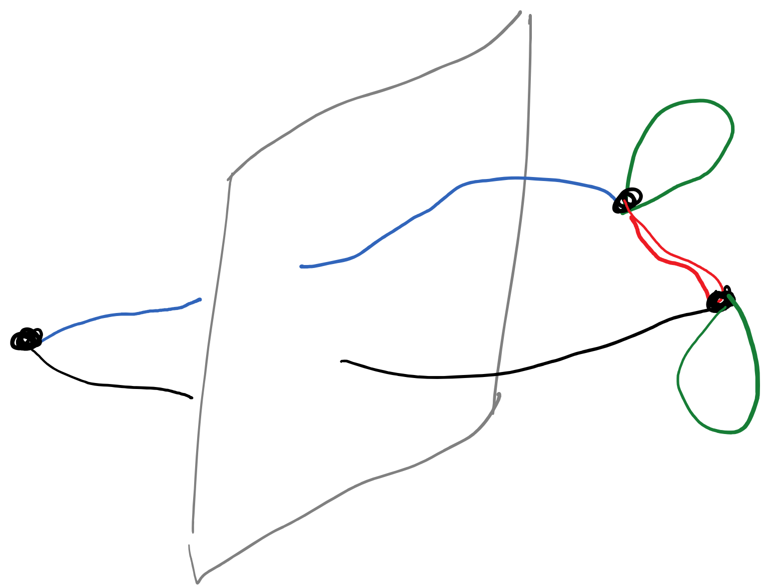
— THIS PROVIDES A PROPOSAL FOR A NOTION OF ENTANGLEMENT IN THE 9+1 DIMENSIONAL GRAVITY THEORY

(S.R.D, A.Kaushal, G.Mandal, S.Trivedi)

THE NON-TRIVIAL EXTENSION INVOLVES THE OFF-DIAGONAL MATRIX ELEMENTS — THE STRINGS WHICH GO BETWEEN DIFFERENT D0 BRANES



THE NON-TRIVIAL EXTENSION INVOLVES THE OFF-DIAGONAL MATRIX ELEMENTS — THE STRINGS WHICH GO BETWEEN DIFFERENT D0 BRANES



WE CONJECTURED THAT

$$S_{EE} = \frac{AREA}{4 G_N}$$

$\Rightarrow$ THE UV SCALE WHICH APPEARS IS G_N RATHER THAN l_s

THIS IS CONSISTENT WITH THE FACT THAT THE MATRIX QUANTUM MECHANICS HAS A MASS SCALE

$$\Lambda = (g_s N)^{1/3} / l_s$$

AND NO OTHER DIMENSIONLESS PARAMETER

NUMERICAL CALCULATIONS CAN BE DONE TO CHECK CONJECTURE

IN FACT THERE IS A MUCH EARLIER VERSION OF SUCH A THEORY WHICH INVOLVES A SINGLE MATRIX

$\Rightarrow$ GAUGED MATRIX QM ,

THIS IS DUAL TO 1+1 DIMENSIONAL STRING THEORY

SINCE A SINGLE MATRIX CAN ALWAYS BE DIAGONALIZED

$$M \longrightarrow \mathrm{diag}\,(\lambda_1, \lambda_2 \cdots \lambda_N)$$

λ_i's BECOME COORDINATES OF N PARTICLES

THE TARGET SPACE ENTANGLEMENT WAS CALCULATED LONG TIME AGO

(S.R. Das, 1995)

AND REVISITED RECENTLY

(S. Hartnoll & E. Mazenc, 2015)

THE RESULT IS FINITE

THE CUTOFF IS PROVIDED BY THE STRING COUPLING

- THIS IS CONSISTENT WITH OUR CONJECTURE THAT IN GENERAL THE CUTOFF IS NEWTON'S CONSTANT