Integral Equations of Yang-Mills Theory

and its Gauge Invariant Conserved Charges

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Main Topics

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with

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Gauge covariant under the transformations

$$\begin{aligned} A_\mu &\rightarrow g\, A_\mu\, g^{-1} + \frac{i}{e}\, \partial_\mu g\, g^{-1} \\ F_{\mu\nu} &\rightarrow g\, F_{\mu\nu}\, g^{-1} \\ J^\mu &\rightarrow g\, J^\mu\, g^{-1} \end{aligned}$$

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Charges

$$Q = \int d^3x\, \partial^i F_{i0} = \int d^3x\, \vec{\nabla} \cdot \vec{E} = \int d\vec{\Sigma} \cdot \vec{E}$$

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→ eigenvalues of Q are gauge invariant

(=) if g is constant at infinity

The Wilson line

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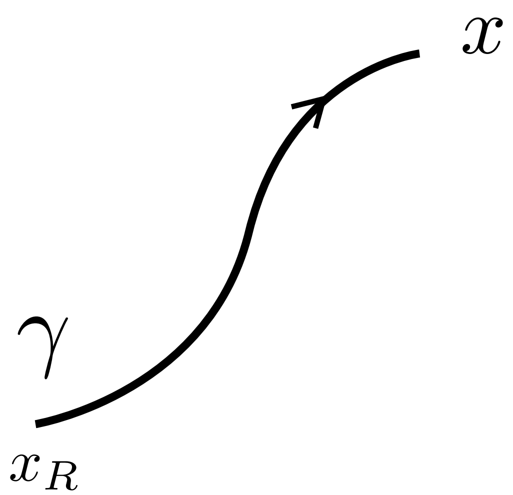

$$\frac{d\,W}{d\,\sigma} + i\,e\,A_\mu\,\frac{d\,x^\mu}{d\,\sigma}\,W = 0$$

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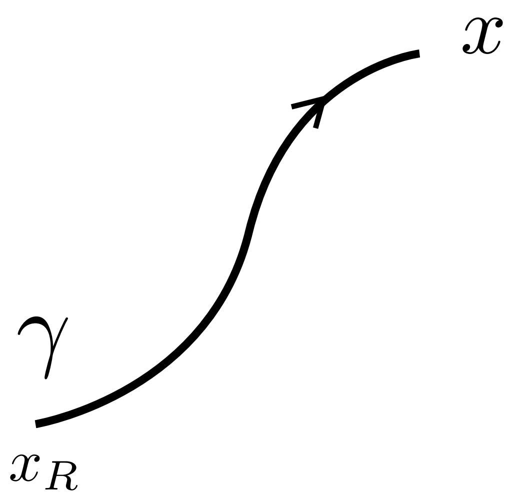

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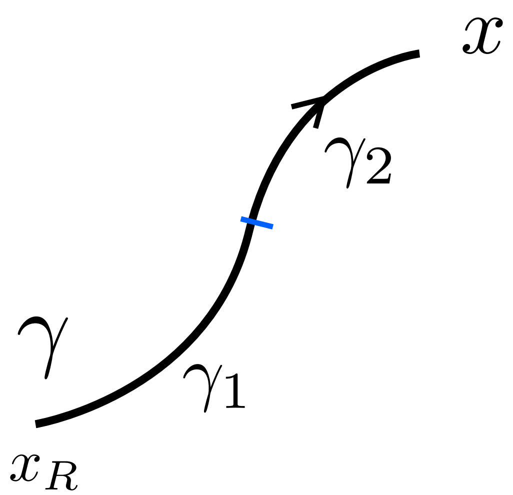

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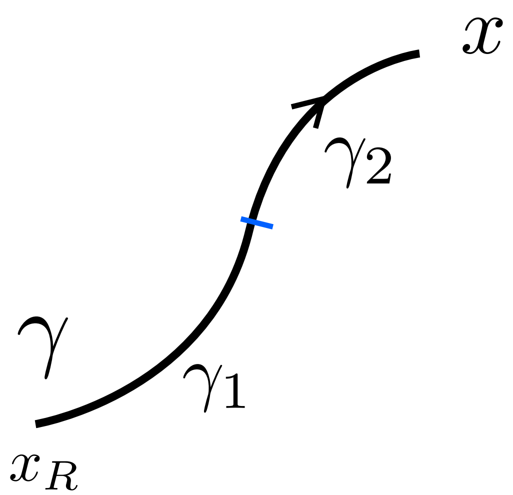

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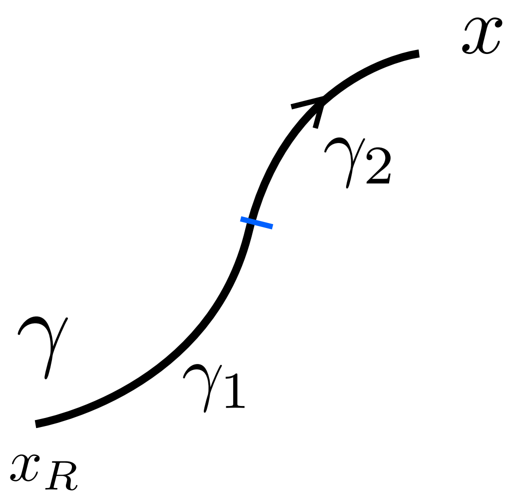

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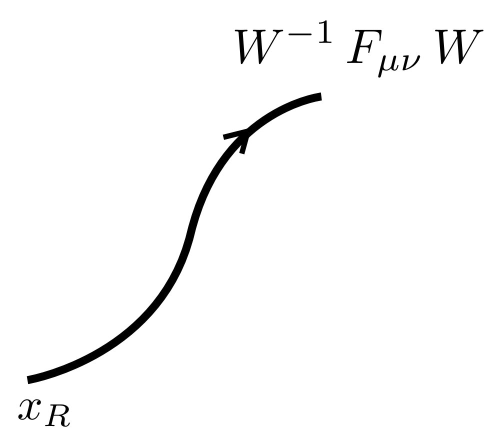

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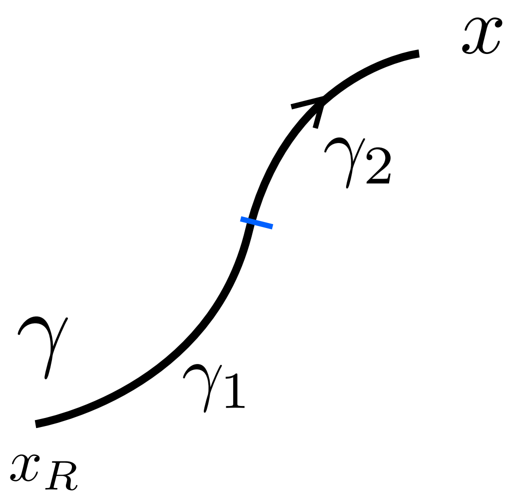

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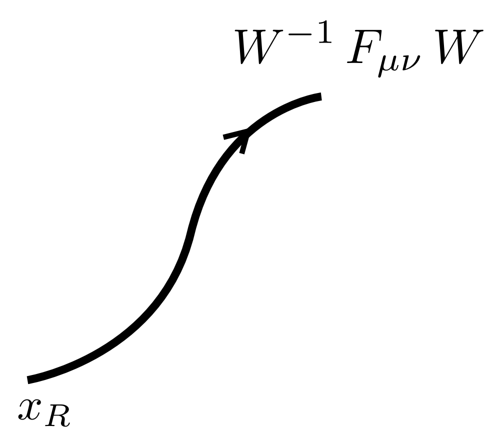

W is path dependent

A non-integrable phase

Wu-Yang and 'tHooft-Polyakov monopoles

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At spatial infinity:

$$A_i = -\frac{1}{e}\,\varepsilon_{ijk}\,\frac{\hat{r}_j}{r}\,T_k \qquad F_{ij} = \frac{1}{e}\,\varepsilon_{ijk}\,\frac{\hat{r}_k}{r^2}\,\hat{r}\cdot T \qquad [T_i\,,\,T_j] = i\,\varepsilon_{ijk}\,T_k$$

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Monopole without a magnetic flux !?

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Wu-Yang needs a source to exist (Constantinidis, LAF, Luchini, JPA (2019))

Gauge Principle

Weyl's non-integrable phase

$$\psi_{\rm int.} = P\, e^{i\, e\, \int A_\mu dx^\mu}\, \psi_{\rm free}$$

Physics is compatible with non-integrable phases

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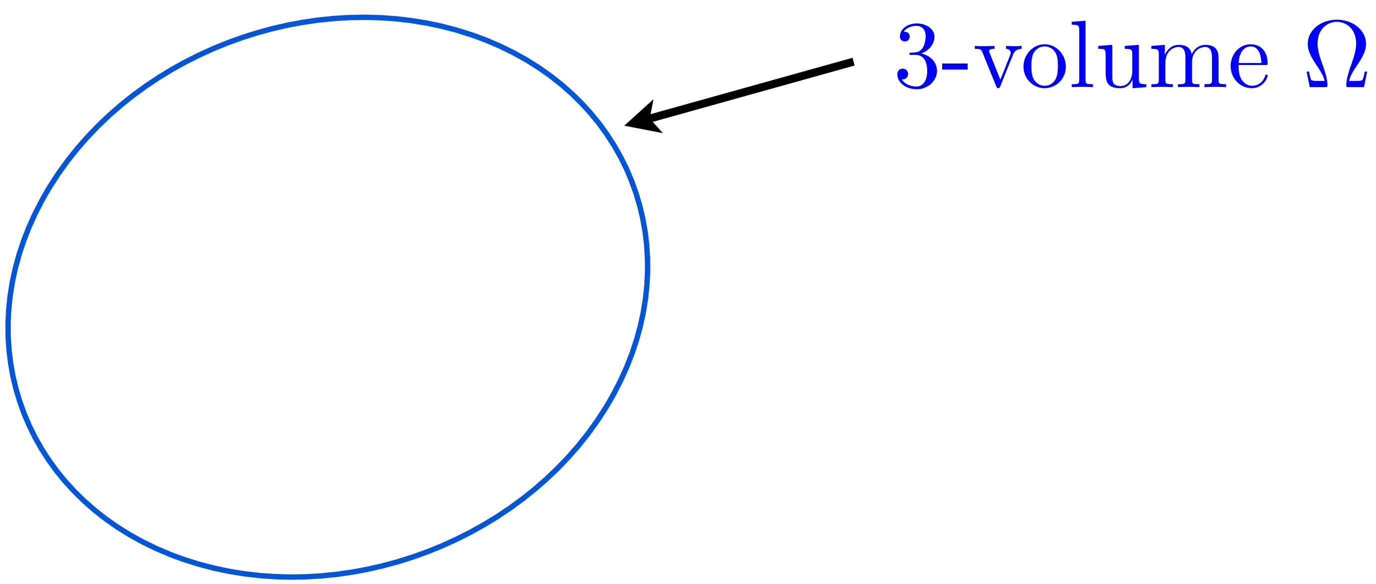

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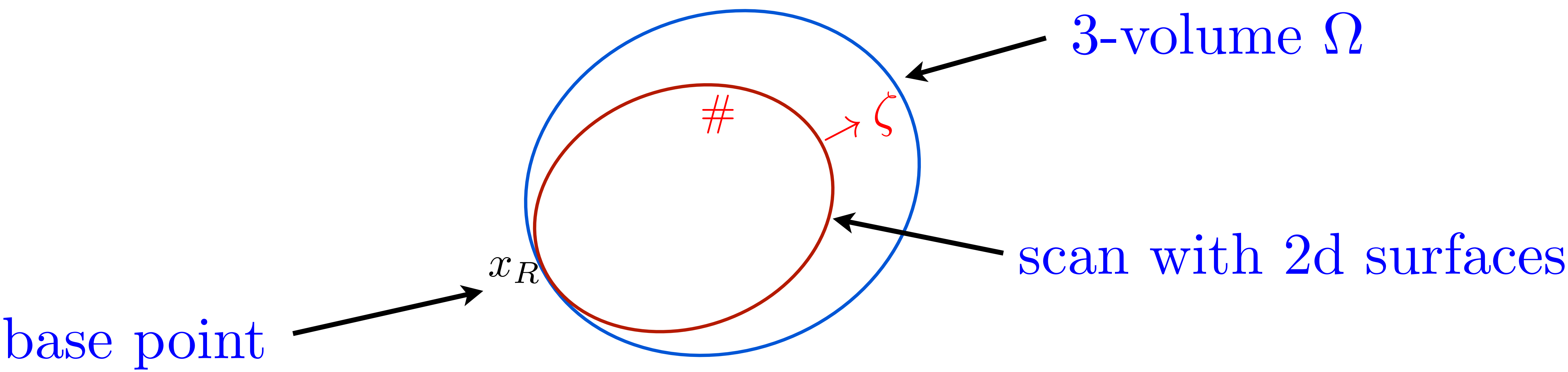

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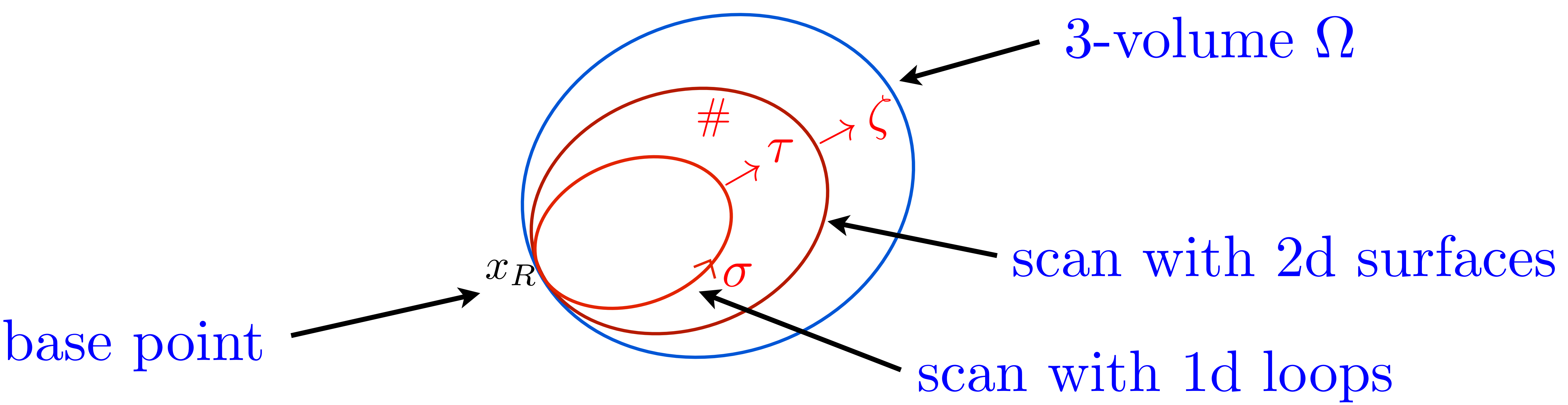

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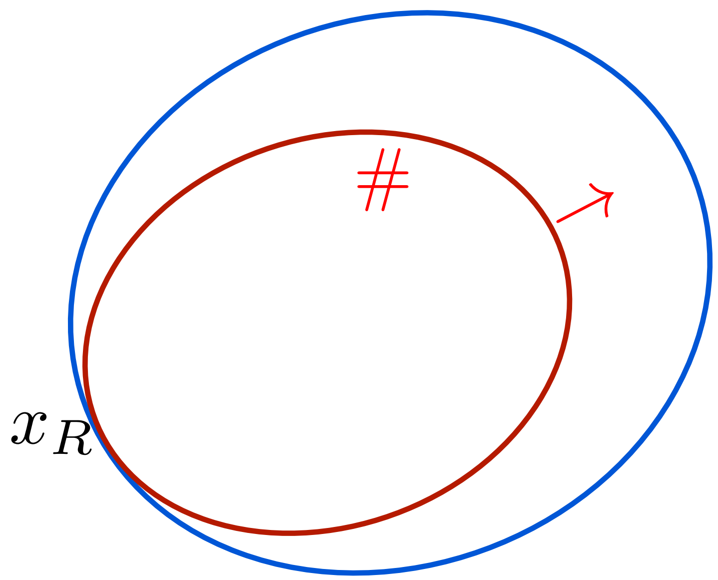

3-volume Ω in M

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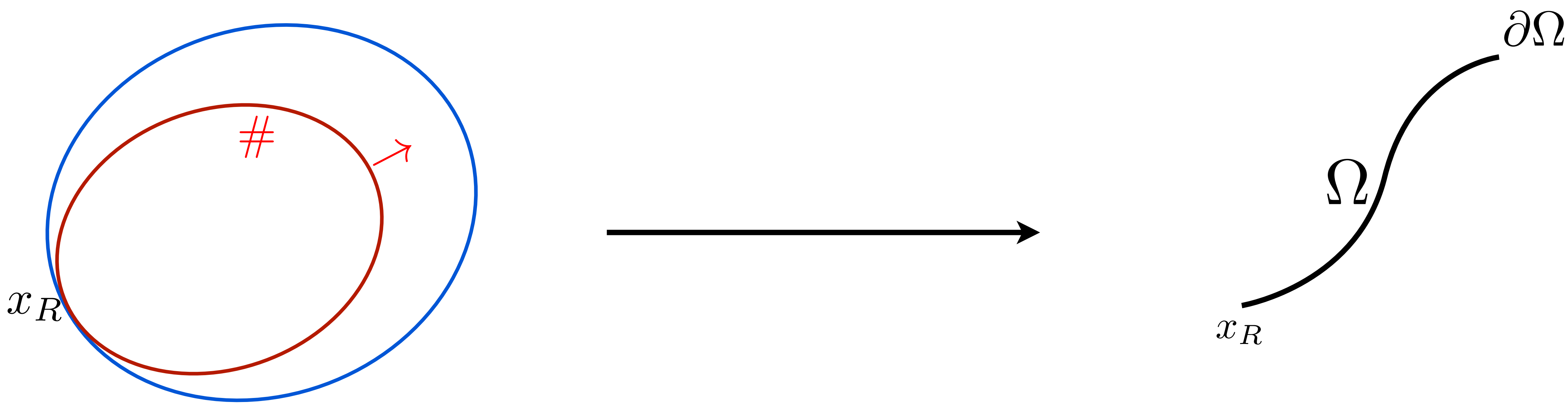

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path in $\mathcal{L}^{(2)}$

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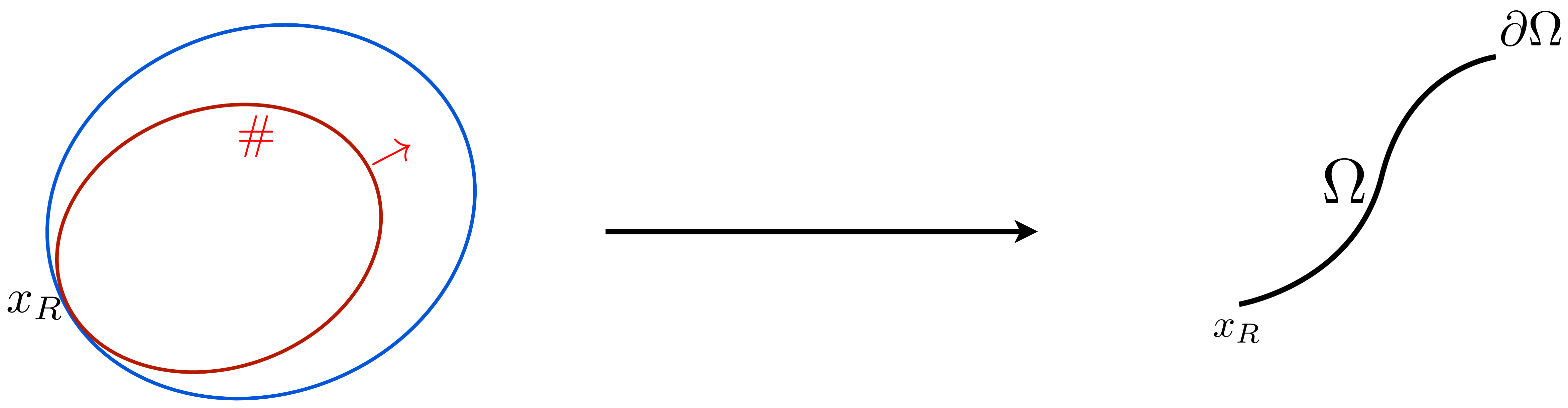

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The right thing: holonomies in loop space

The generalized non-abelian Stokes theorem

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$$\frac{d\,V}{d\,\tau} - V\,T\,(A, B, \tau) = 0$$

$$T\,(B, A, \tau) \equiv \int_0^{2\,\pi} d\sigma\; W^{-1}\, B_{\mu\nu}\, W\, \frac{d\,x^\mu}{d\,\sigma}\, \frac{d\,x^\nu}{d\,\tau}$$

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$$\begin{aligned}
\mathcal{K} \equiv & \int_0^{2\,\pi} d\tau \int_0^{2\,\pi} d\sigma\, V\,(\tau)\; \Big\{ \\
& W^{-1}\, \left[D_\lambda B_{\mu\nu} + D_\mu B_{\nu\lambda} + D_\nu B_{\lambda\mu}\right] W \frac{d\,x^\mu}{d\,\sigma}\, \frac{d\,x^\nu}{d\,\tau}\, \frac{d\,x^\lambda}{d\,\zeta} \\
& - \int_0^\sigma d\sigma' \left[\, B^W_{\kappa\rho}\,(\sigma') - ieF^W_{\kappa\rho}\,(\sigma')\,,\, B^W_{\mu\nu}\,(\sigma)\,\right] \frac{dx^\kappa}{d\sigma'}\, \frac{dx^\mu}{d\sigma} \\
& \times \left(\frac{d\,x^\rho\,(\sigma')}{d\,\tau}\, \frac{d\,x^\nu\,(\sigma)}{d\,\zeta} - \frac{d\,x^\rho\,(\sigma')}{d\,\zeta}\, \frac{d\,x^\nu\,(\sigma)}{d\,\tau} \right) \Big\}\, V^{-1}\,(\tau)
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The integral equations for Yang-Mills

The integral equations for Yang-Mills

$$P_2 e^{ie\int_{\partial\Omega} d\tau d\sigma \left[\alpha F^W_{\mu\nu} + \beta \widetilde{F}^W_{\mu\nu}\right] \frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\tau}} = P_3 e^{\int_\Omega d\zeta d\tau V \mathcal{J} V^{-1}}$$

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Direct consequence of Stokes theorem and Yang-Mills eqs.

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Implies Yang-Mills eqs. in the limit $\Omega \to 0$

Conserved Charges

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If Ω_c is a closed volume $(\partial\Omega_c = 0)$ $\longrightarrow$ $P_3 e^{\oint_{\Omega_c} d\zeta d\tau V \mathcal{J} V^{-1}} = \mathbb{1}$

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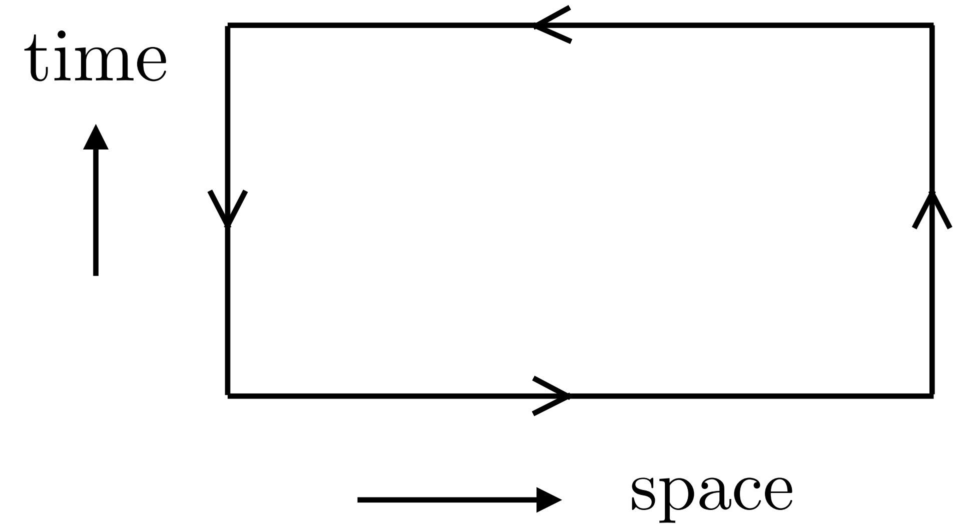
A closed volume in M is a closed path in $\mathcal{L}^{(2)}$



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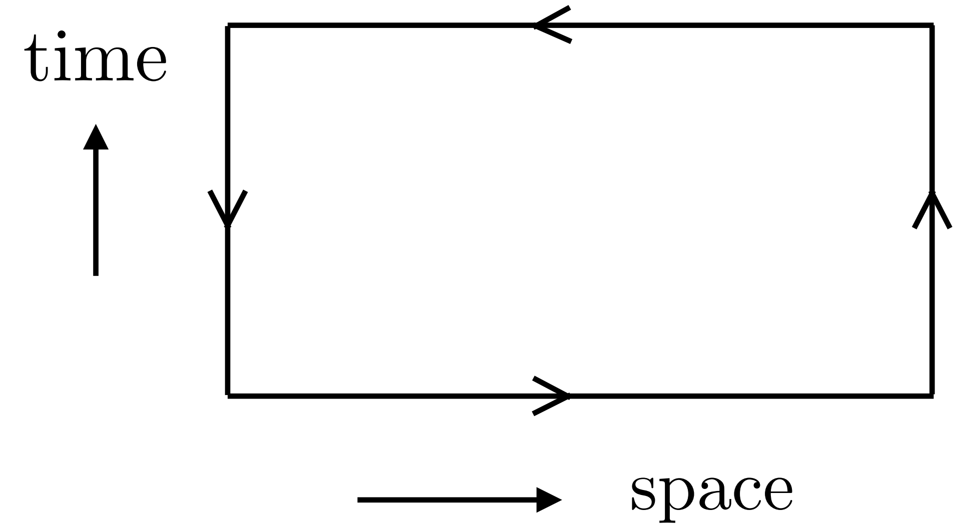

Iso-spectral evolution:

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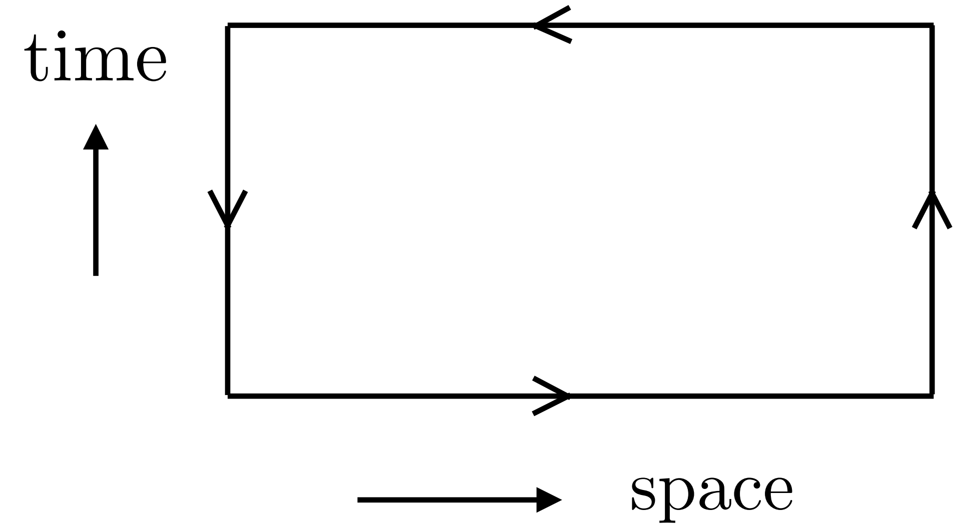

Iso-spectral evolution: $$V(\Omega_t) = U(t) \cdot V(\Omega_0) \cdot U^{-1}(t)$$

Eigenvalues of $V(\Omega_t)$ are constant in time

Conserved charges are eigenvalues of the operator

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5) Infinite number of charges (expansion in α and β)

C.P. Constantinidis
G. Luchini

Thank You

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