Entropía De Entrelazamiento

Luis Carlos Desa Salas, 170735@unsaac.edu.pe

Escuela Profesional De Física, Universidad Nacional De San Antonio Abad Del cusco , Perú

Introduction

In the current problem of the information paradox, it has led to generate new proposals for the entropy of black holes, in which the Hawking-Bekenstein entropy is incomplete, so a "generalized" entropy is proposed that has a extra term that considers quantum fields outside the event horizon [1]

$$S_{gen} = \frac{\mathcal{A}_{HS}}{4\hbar G_N} + S_{outside} \tag{1}$$

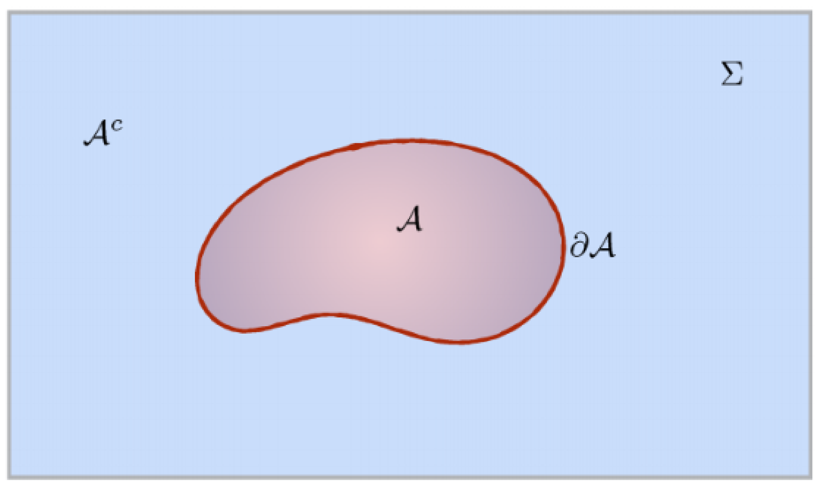
This proposal is based on the entropy proposed by Ryu-Takanayagi, this is the entropy of entanglement.

Entanglement Entropy

The entangled state is defined as a whole, by the non-separability of the parts, therefore the pure quantum state belongs to the tensor product of the Hilbert space of the two subsystems, which are separated by the entanglement surface

$$|\psi\rangle \in H = H_A \otimes H_{A^c} \tag{2}$$

$$|\psi\rangle = \sum_{i,j} c_{ij} |i\rangle_A \otimes |j\rangle_{A^c} \tag{3}$$



In order to calculate the entanglement entropy we use the von Neumann equation, for which we need the reduced density matrix[3]

$$\rho_A = Tr_{A^c}(|\psi\rangle\langle\psi|) \tag{4}$$

$$S_A = -Tr_A(\rho_A \log \rho_A) \tag{5}$$

If the reduced density matrix is diagonalizable

$$S_A = -\sum(\lambda_i \log \lambda_i) \tag{6}$$

Continuum QFTs

To calculate the entanglement entropy of continuous fields, we must make a temporary slice $(t=0)$, that for a d-dimensional QFT, (Σ_{d-1}), the reduces density matrix will be a path integral with the boundary conditions of the past and future causal of our temporal slice

$$(\rho_A)_{-+} = \int [D\Phi] e^{-S_{QFT}[\Phi]} \delta_E(\Phi_{\mp A}) \tag{7}$$

$$\delta_E(\Phi_{\mp A}) = \delta(\Phi_A(t=0^- - \Phi_-))\delta(\Phi_A(t=0^+ - \Phi_+)) \tag{8}$$

Entanglement Entropy of CFT_3

With the considerations mentioned above, we can obtain the following result for a d-dimensional QFT, where the first term is the area law, it is required that$(d \geq 3)$ [2]

$$S_{EE}^{QFT} = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_1 \frac{R^{d-4}}{\delta^{d-4}} + \ldots + a \log \frac{R}{\delta} - F \tag{9}$$

For a circumference of radius R, that is $(d=3)$, we obtain the following result, (it can also be for non-smooth surfaces)

$$S_{EE} = c_0 \frac{R}{\delta} - F \tag{10}$$

Where (δ) is a k-top, a scale constant.

Holographic Entanglement Entropy

Inspired by the area law, the following equation is proposed to calculate the holographic entanglement entropy

$$S_{EE}(A) = \frac{minArea(\gamma_A)}{4G_N} \tag{11}$$

Where (γ_A) is a surface that covers the circumference and extends into spacetime AdS_4, the minimum value of this surface is used, then $(Area(\gamma_A))$ is an extremal. For the calculation we will use the Poincare Patch metric

$$ds_{PP}^2 = \frac{L^2}{z^2}\left[-dt^2 + dz^2 + dr^2 + r^2 d\theta^2\right] \tag{12}$$

With $z = f(r)$ and $\dot{f}(r) = df(r)/dr$, how $(t=0) \Rightarrow (dt=0)$

$$ds_\gamma^2 = \frac{L^2}{f(r)^2}\left[(\dot{f}(r)^2 + 1)dr^2 + r^2 d\theta^2\right] \tag{13}$$

With the determinant of its metric $(det(h) = \sqrt{h})$

$$\sqrt{h} = \frac{L^2}{f(r)^2} r \sqrt{1 + \dot{f}(r)^2} \tag{14}$$

The entanglement entropy

$$S_{EE}(A) = \frac{1}{4G} min \int \sqrt{h} dr d\theta \tag{15}$$

$$S_{EE}(A) = \frac{1}{4G} min \int \frac{L^2}{f(r)^2} r \sqrt{1 + \dot{f}(r)^2} dr d\theta \tag{16}$$

Applying the Euler-Lagrange equations, inside the integral, the function $f(r)$ that satisfies it is $(z = f(r) = \sqrt{R^2 - r^2})$, replacing

$$S_{EE}(A) = \frac{\pi L^2}{2G} \int_0^R \frac{r}{(R^2 - r^2)^{3/2}} dr \tag{17}$$

Solving the integral, we obtain the following result, which is the same as the one obtained previously, with a quantum calculation (10).

$$S_{EE}(A) = \frac{\pi L^2}{2G}\frac{R}{\delta} - \frac{\pi L^2}{2G} \tag{18}$$

$$S_{EE}(A) = c_0 \frac{R}{\delta} - F \tag{19}$$

Conclusions

- Thus we can conclude that the holographic calculation with the equation (11) for the entanglement entropy is much more efficient than from a quantum theory CFT.

References

[1] M. S. T. Almheiri, Hartman. The entropy of hawking radiation, 2020. URL arXiv:2006.06872v1[hep-th].

[2] C. Calabrese. Entanglement entropy and conformal field theory, 2009. URL arXiv:0905.4013v2[cond-mat.stat-mech].

[3] T. Rangamani. Holographic entanglement entropy, 2016. URL arXiv:1609.01287v2[hep-th].